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Three essays in macroeconomics

by

Xue Qiao

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Economics

Program of Study Committee:
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Iowa State University

Ames, Iowa

2007

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ABSTRACT

This dissertation is concentrated on two different economic topics: unemployment insurance (UI) and longevity. The first two chapters examine how a government UI program affects individuals' behavior and the labor market performance. The last chapter investigates the role of the interaction between public health expenditure and private health investment in promoting a longer lifetime span.

In the first chapter, I study the effects of a publicly funded unemployment insurance (UI) system on a firm's decision to terminate employment contracts in a dynamic moral hazard model. It is shown that the optimal employment contract involves termination of both sufficiently rich (too rich to motivate with compensation) and sufficiently poor (too poor to punish) workers. Unemployment insurance, by reducing the termination cost to firms, reduces the cut-off wealth level at which it becomes optimal to fire workers. When calibrated to the U.S. data, the model suggests that the presence of an unemployment insurance system may cause more job terminations but may also reduce the unemployment rate. While consumption of unemployed agents is shown to be changed little, aggregate welfare of all workers is reduced. The upshot is that a firm that offers long-horizon employment contracts can provide substantial consumption insurance and a publicly funded UI system would only crowd out the firm's insurance provision.

The second chapter evaluates the effects of the current unemployment insurance (UI) program in a model where agents are able to borrow and lend, and the government holds an unemployment insurance trust fund (UITF) balance. The only borrowing constraint in this economy is that an agent's consumption must be non-negative. It appears that the current positive UITF balance improves the welfare by 0.65%, relative to the economy with a UITF

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balance of 0 and yet the existing UI program is still available. In addition, removing the current UI program from the economy improves the welfare by 1.79%, which suggests that the public UI program may not be needed when agents are able to smooth consumptions by borrowing.

The last chapter introduces endogenous longevity in an otherwise standard overlapping generations model with capital. In the model, a young agent may increase the length of her old age by incurring investments in health funded from her wage income. Such private health investments are assumed to be more "productive" if accompanied by complementary tax-financed public health programs. The presence of such a complementary public input in private longevity is shown to expose the economy to aggregate endogenous fluctuations and even chaos, and such volatility is impossible in its absence.

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CHAPTER 1. GENERAL INTRODUCTION

1.1 Introduction

This dissertation is concentrated on two different economic topics: unemployment insurance (UI) and longevity. I first examine how a government UI program affects individuals' behavior and the labor market performance. Next, I investigate the role of the interaction between public health expenditure and private health investment in promoting a longer lifetime span.

In U.S., unemployment insurance is a federal-state joint program. It was first created in 1935 in response to the Great Depression when millions of people lost jobs. The purpose of this program is to provide temporary income support for laid off workers, and hence, it helps cushion the impact of economic downturns. There are two main features of the government UI program: benefits and taxes. In general, the program is financed by employer payroll taxes and makes unemployment benefits to eligible workers who are unemployed through no fault of their own. More specifically, an eligible unemployed worker can receive a percentage of his previous earnings up to 26 weeks.

Although the UI program helps workers to better smooth their consumption during bad times, it may also generate some side effects by affecting individuals' job searching behavior, firms' layoff decisions, and the labor market performance. A large number of the literature have been focused on the perspective of workers' behavior. They have shown that, given the unemployment insurance benefits, an unemployed worker may reduce his job-searching intensity. As a result, the UI program may induce a higher unemployment rate, which further increases the UI benefit claims. Along with this direction, many recent studies consider the optimal UI design problem in the context of moral hazard. Other than these studies, there are some other work which study the UI effects from a different angle, that is, how the UI

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affects a firm's layoff decision. The layoff is induced by an exogenous shock, for example, a negative demand shock or a structural reform. In other words, layoff deicision is exogenously imposed. However, in practice, we also observe the existence of unemployments when there is no exogenous shock. And it is very possible that the UI program may affect these types of layoff deicisions, or, let's say, endogenous layoff deicisions.

The second chapter of my dissertation considers how the UI program affects a firm's decision to layoff workers and then the aggregate economic performance through that channel. In this chapter, an agent's effort is private information, i.e., moral hazard problem exists. Workers are laid off because they are either too poor to be punished or too rich to be motivated. Here, layoff decisions are endogenously determined by firms. And this is the key feature that differs my paper from previous work.

My third chapter departs from the first chapter by switching back to workers. As mentioned earlier, the UI program can help agents smooth their consumption, however, in the cost of distortorting their behaviors. Thus, the magnitude of the benefits and costs are important in evaluating the UI program. Nevertheless, in addition to the UI program, agents can also self-insure themselves against the unemployment risk, by using savings or borrowing to smooth the consumption. Hence, the magnitude of the UI effects is largely depending on the extent of the self-insurance scheme. And this is more or less limited in the previous research work. For instance, some do not allow agents to save and borrow at all, and others allow agents to save, but only in the form of non-interest-bearing assets, or storage. All those limitations are relaxed in the current work. Agents are allowed to borrow and lend, as long as their consumptions are non-negative. Additionally, the government budget is not zero-balanced anymore. Or equivalently, the government can hold a balance in the unemployment insurance trust fund (UITF) account. This adds in another channel for the government to affect workers' behavior through the interest rate. It turns out the avilability of the self-insurance scheme does matter significantly in measuring the UI effects. In contrast to the previous findings, my work shows that the existing UI program reduces the welfare.

The fourth chapter of my dissertation is a joint work with my major professor, Dr. Joydeep

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Bhattacharya. It studies a totally different topic from the previous two, that is, the longevity. Data have suggested that there exists a large difference in life expectancy across countries. For example, the gap between the highest ten life expectancy countries and the lowest ten is almost 40 years. Given this fact, it seems natural to ask what is causing this huge differences. The world health organization argues that this difference has a lot to do with the economics of the health system.

Health system has two components: a public health sector and a private one. The private health sector refers to the effort made by individuals to improve health and longevity, while the public health system works in a way to promote a healthy environment, and healthy behaviors for the whole population. Since one's own longevity is not only affected by his own health behavior, but also by others who are around him, it seems reasonable to assume that a larger public health expenditure complements the private health investment made by individuals. Our work present a theoretical framework to study this complementarity between public health programs and private health effort to improve longevity. The important finding is that tax-financed public expenditures aimed at enhancing individual agent longevity may introduce endogenous volatility in an economy where such fluctuations are impossible in their absence.

1.2 Thesis Organization

This dissertation contains five chapters. Chapter 1 introduces the general background for three papers and the scope of the dissertation. Chapter 2 presents the model for the endogenous layoff, calibrates the model parameters to replicate the U.S. data and compares the equilibrium outcomes for several policy experiments. Chapter 3 studies the effects of the existing UI system in an environment where agents are able to borrow and lend, and the government holds a possible non-zero balanced budget. In addition, it conducts two policy experiments. One is to remove the existing UI system and the other is to set the government budget to be zero-balance. Chapter 4 builds up a theoretical framework to study how the complementarity between the public health programs and private health investment affects

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longevity. The summary of the conducted work and overall conclusions of the studies are presented in Chapter 5.

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CHAPTER 2. UNEMPLOYMENT INSURANCE IN A DYNAMIC ECONOMY WITH ENDOGENOUS LAYOFFS

2.1 Abstract

This paper studies the effects of a publicly funded unemployment insurance (UI) system on a firm's decision to terminate employment contracts in a dynamic moral hazard model. It is shown that the optimal employment contract involves termination of both sufficiently rich (too rich to motivate with compensation) and sufficiently poor (too poor to punish) workers. Unemployment insurance, by reducing the termination cost to firms, reduces the cutoff wealth level at which it becomes optimal to fire workers. When calibrated to the U.S. data, the model suggests that the presence of an unemployment insurance system may cause more job terminations but may also reduce the unemployment rate. While consumption of unemployed agents is shown to be changed little, aggregate welfare of all workers is reduced. The upshot is that a firm that offers long-horizon employment contracts can provide substantial consumption insurance and a publicly funded UI system would only crowd out the firm's insurance provision.

2.2 Introduction

Millions of workers encounter unemployment spells over the course of their lives. If they could, sufficiently risk-averse workers would purchase private insurance to finance their consumption needs during such periods of joblessness. However, as is well known, insurance markets are incomplete and private unemployment insurance cannot be purchased. In its absence, workers may save to smooth their consumption across periods of employment and unemployment, but often their savings are insufficient for that purpose. A publicly (government) funded

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unemployment insurance (UI) is an attempt to solve this problem.[1]

In most developed economies, UI programs funded by governments do a decent job in their coverage [2] and in the amounts of the benefits paid to unemployed workers.[3] However, there are unintended and possibly undesirable effects on workers, too. By its action, UI reduces the opportunity cost of unemployment, thereby reducing search effort and increasing both the duration of unemployment spells and the equilibrium rate of unemployment. This paper focuses on the inadvertent effects of UI on firm behavior, especially those effects concerning firms decisions to terminate employment contracts.

In doing so, this paper breaks away from most of the literature by shifting the focus away from the effect of UI on workers onto the effect of UI on firms. This shifting of emphasis seems natural. After all, firms (and not the government) lay off the workers that the government insures, but the government has no direct control over the firms. Nevertheless, as I show in this paper, the indirect effect of the government-controlled UI system on firm behavior can be qualitatively and quantitatively important. Indeed, my setup allows me to study the interaction between a firm's provision of insurance (via severance payments) and that provided by the government via the UI system. The next novel feature of the paper is its derivation of optimal dynamic employment contracts between firms and workers in the presence of public UI. Unlike most other papers in the literature, the employment relationship is not modeled as a series of one-period contracts, but rather as a long horizon contract. This focus on long term employment contracts is shown to have important implications for the amount of insurance that the firm can provide the worker.[4] Finally, in contrast to most of the earlier work, the

[1]In the US, the unemployment insurance program was created in 1935 to respond to the Great Depression. It is a Federal-State joint program whose purpose is to "provide temporary financial assistance to unemployed workers who meet the requirements of State law." In the majority of the states, benefit funding is based solely on a tax imposed on employers.

[2]Blank and Card (1991) find that, for the CPS sample 1977-1987, on average, 30.4% of the unemployed collect UI benefits. Also, according to a report from the Government Accountability Office (GAO), UI covered about 129 million wage and salary workers and paid about $41 billion in benefits to nearly 9 million workers who lost their jobs in fiscal year 2004.

[3]In general, UI benefits are based on a percentage of an individual's earnings over a recent 52-week period and up to a state maximum amount. It can be paid for a maximum of 26 weeks in most states.

[4]In a standard model of moral hazard with one period contracts, the firm has very little power to provide any insurance; in any period, it can either reward the employee a lot for high effort/output or punish him severely for bad performance. With dynamic contracts, the firm could spread the rewards and the punishments more evenly over time, thereby offering more insurance to the worker.

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paper presents a first attempt at studying the general equilibrium effects of UI in a model where termination of employment contracts is endogenously derived.

Specifically, this paper studies a fully dynamic model of the labor market along the lines of Wang (2005) where long-term employment contracts are terminated optimally. In the model economy, there is a continuum of firms and workers that are each ex-ante identical. Workers are risk averse. There are no assets. I assume a firm can be matched with only one worker at any time. Once a firm and a worker are matched, production of the single perishable good takes place. The only input is effort and output is a random variable. There is moral hazard because the workers' efforts are not observable. In each period, the firm observes the worker's realized output, provides compensation to the worker, and decides to continue or terminate the employment relationship. When a worker's employment contract is terminated, the firm can make a severance payment to the worker. The firm then returns to the labor market to seek a new match. The UI benefit to a worker is a fraction of his expected consumption at his last place of employment and is financed by a lump sum tax on all firms.

Conditional on being terminated, a worker makes his decision to enter or exit the labor market. If he enters the labor market, he is eligible for the UI benefit; otherwise, not. An agent exits the labor market permanently if the offered expected utility conditional on reentering the labor market is less than or equal to his current utility. This comes from the fact that the firm has perfect information about the UI system, and hence, knows the exact amount of unemployment benefit a worker can claim if terminated. This makes a firm's termination decision dependent on the amount of unemployment benefit.

The details of the employment contract are given below. In my setup, the firm motivates effort by rewarding the worker with high compensation in the high output state, and by punishing him with low compensation if low output occurs. Even though workers are ex-ante identical, their different output histories will create differences in the expected lifetime utilities promised by firms. As in Spear and Wang (2005), two types of terminations emerge. In one, an agent is fired because she has experienced a long sequence of bad outputs, and hence, becomes too impoverished to be punished. In the other, an agent who has experienced a long sequence

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of high outputs is too rich and, hence, becomes too expensive to motivate.

I calibrate the model to the US economy and study three sets of issues. First, I study how a firm's termination behavior is affected by the current UI system. Secondly, I compute the effects of the current UI system on aggregates such as the levels and flows into and out of unemployment, employment and retirement. Lastly, I compare aggregate welfare across economies with and without the current UI system. The major quantitative findings are as follows: The current unemployment insurance induces more job termination and more workers to exit the labor market; with the current UI, the rate of unemployment is reduced from 7.26% to 6.47%; consumption of unemployed workers is not changed much by the current unemployment insurance, however, the welfare of all agents in the economy is reduced.

The result that UI reduces the equilibrium rate of unemployment seems surprising, conventional wisdom being that UI lowers workers' searching intensity and, therefore, increases the unemployment rate. UI affects the labor market in a very different way in this model. In my model, UI creates more job vacancies and causes more workers to exit the labor force permanently, which combine to bring down the equilibrium unemployment rate.

First, UI induces more job termination. There are two effects. Firms use termination as an incentive device to motivate effort. Since the introduction of the UI program reduces firms' costs to lay off workers with poor performance, termination will be used more frequently as a punishment device. On the other hand, since the payment of the UI benefit makes layoff a softer punishment to the worker, more terminations must be used in order to implement a given amount of punishment.

Second, UI lowers the size of the labor force. Since the UI tax is imposed on all producing firms, it has a distortionary effect in the sense that the value of the firms, especially those with "rich" workers (due to a sequence of high outputs) is reduced. As a result, firms terminate rich workers more often, resulting in more retirements in the equilibrium. And thus, the number of unemployed workers decreases.

An important thing to note here is that the firm (via its employment contract) and the government (via the UI system) are both insuring the worker; indeed, the government crowds

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out the insurance offered by the firm. As a result, the consumption level of the jobless is much the same in the presence or absence of the UI system. Overall welfare may be lower, since the workers who leave the labor force do so with lower expected utilities. The upshot is that if firms can offer dynamic employment contracts, a publicly funded UI system may be unnecessary from the point of view of worker utility.

My paper is part of a long line of research studying the UI system using dynamic micro-founded models. It is probably safe to say that most of the theoretical research in this area has focused on the question of optimal UI design in the context of moral hazard. The seminal paper is Shavell and Weiss (1979) who study the optimal sequencing of benefits (from the point of view of unemployed workers) in a partial equilibrium model where the UI benefit per unemployed worker is fixed and workers' searching effort is not observable. In their model, workers only experience a single unemployment spell. They show that the optimal UI benefit should decline monotonically over time. Hopenhayn and Nicolini (1997) study the same problem in the same environment with an additional policy instrument: a wage tax after reemployment. They show that this tax increases with the length of the unemployment spell. Hopenhayn and Nicolini (2005) then look at the optimality of employment history related restrictions [5] in a model with multiple unemployment spells, where the principal cannot distinguish "quits" from layoffs.

The paper that is possibly closest in spirit to mine is Wang and Williamson (1996), in the sense that they study the problem of optimal UI design in a general equilibrium model where job termination is endogenous. They add another source of moral hazard: an agent's employment status depends on his own effort. They find that an optimal UI system involves a large subsidy to those who move from being unemployed to being employed and a large tax for the reverse movement.

A number of papers consider the optimal UI design when agents are allowed to borrow and save. Costain (1997) develops a model of UI with endogenous search intensity and precautionary saving. Werning (2002) considers a problem similar to that of Hopenhayn and Nicolini, where an agent's savings is hidden information.

[5] One example used by Hopenhayn and Nicolini is that, in the US, a minimum of six months of employment is needed in order to qualify for benefits.

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In contrast to many of the previously discussed papers, my focus is not on the optimal design of UI systems. Indeed, in my setup, the firm alone provides consumption insurance to the worker in the form of severance payments, without any need for the institution of UI. Second, and perhaps more importantly, I focus on the effects of the UI system on firms' termination behaviors. My work is related to Feldstein (1978), in the sense that we both focus on how the current UI system affects a firm's layoff decision. Feldstein focuses on temporary layoffs induced by cyclical fluctuations in demand and shows analytically that the current system of unemployment insurance provides a substantial incentive for increased temporary layoff unemployment. The result, that the current unemployment insurance system induces more job termination, is somewhat similar to the one derived in Feldstein (1978).

Section 2.3 presents the model and the definition of a stationary equilibrium. Section 2.4 describes the computation method and the features of the calibrated solution. Section 4.1 compares the steady state solution with different replacement rates, and Section 4.2 examines the effects of UI systems for two environments with different output risks. Section 5 concludes.

2.3 Model

2.3.1 Environment

In this section, I describe a variant of Wang's general equilibrium model, with unemployment insurance embedded in it. Firms now make decisions based upon their expectations of the labor market and the UI insurance system.

Time is discrete and lasts forever. There is only one perishable consumption good in the economy.

The economy has a sequence of overlapping generations of workers. The total measure of the workers in the economy at any time is 1. Each worker survives into the next period with a probability Δ. At the beginning of each period, a measure of $1-\Delta$ new workers are born. An agent who is born in period ϵ has the following lifetime expected utility: $E_\epsilon \sum_{t=\epsilon}^{\infty} (\beta\Delta)^{t-\epsilon}[u(c_t) - \phi(a_t)]$, where $\beta \in (0,1)$ is the discount factor, $c_t \in \Re_+$ denotes period t consumption, and $a_t \in \Re_+$ denotes effort level. I assume $u' > 0$, $u'' < 0$, $\phi' > 0$, $\phi'' > 0$, and $\phi(0) = 0$.

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There are $\eta \in (0,1)$ number of identical firms. Each firm lasts forever and maximizes its expected discounted profits. I assume a firm can be matched with only one worker at any time. Once a firm and a worker are matched, production takes place. The only input in the production is effort. The output of the production is a random variable $\theta \in \Theta = \{\theta_1, \theta_2, ..., \theta_n\}$, where $\theta_i < \theta_j$ for $i < j$. If the employed worker makes effort a, the realized output takes value θ_i with probability $\pi_i(a)$, where $\sum_{i=1}^{n} \pi_i(a) = 1$.

There is a government in the economy. Each period, the government imposes a lump sum tax τ on each firm in the economy. The tax revenue is used to make unemployment insurance payments to the unemployed workers. I assume an unemployed worker is eligible to receive UI benefits only when the following three conditions are satisfied: (1) he is employed prior to his unemployment; (2) he must be active in the labor market; (3) he can not refuse job offers. Given that an unemployed worker is eligible for UI benefits, the UI benefit he can receive is a ratio, b, of his expected compensation when he was last employed. I also assume that an unemployed worker can claim the UI benefit forever.[6]

2.3.2 Contract

There are two activities in the labor market: production and job matching. I call a firm and a worker who are matched together a matched pair. At the beginning of a period there are matched pairs, and unmatched firms and workers. Matched pairs are in the process of production and do not engage in labor market activities. With a probability, an unmatched firm and worker are matched. They exit the labor market and engage in a potentially long-term contractual relationship. If a worker and a firm are not matched, they stay in the labor market and proceed to the next period. Consider a pair that is just matched. The employed worker makes an unobservable effort and output is realized. The firm observes the realized output, makes current compensation to the current worker, and decides whether to retain or terminate him. Conditional on being terminated, a worker makes his decision to enter or exit

[6]Adding another dimension for the periods of unemployment inevitably makes the model almost impossible to study. But I admit that allowing a worker to claim UI benefits forever will exaggerate the effects of UI on the labor market.

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the labor market. If he enters the labor market, he is eligible for unemployment insurance benefits; otherwise, he is not. Note that since the worker is risk averse, he may be able to receive a termination package that delivers a sequence of constant payments in the future.

Following Green (1987) and Spear and Srivatana (1987), I define V as the expected utility of a worker at the beginning of a period. When there is no UI system, a firm considers its profit maximization problem based only on a worker's current status. Therefore, V is enough to identify a state. This is not the case when unemployment insurance is added. The insurance benefit an unemployed worker is able to receive is based on his earnings prior to unemployment. The firm has perfect information about the UI system, and hence, knows the exact amount of the unemployment benefit a worker can claim if he is terminated. Therefore, unemployment insurance also enters into a firm's problem. That is, a firm's termination decision now depends on the amount of the unemployment benefit. To address this concern, I add in another variable, $\tilde{V}$, a worker's expected utility at the beginning of the period last employed. Given that, a contract σ can now be written as:

$$\sigma = \{a(V), c_i(V), V_i(V)\ \forall V \in \Phi_h\},$$

for the initial period of the contract and

$$\sigma(V) = \left\{ \begin{array}{ll} \Phi_r(\tilde{V}) \cup \Phi_f(\tilde{V}) = \Phi & \\ a(V), c_i(V), V_i(V) & \forall V \in \Phi_r(\tilde{V}) \\ g(\tilde{V}, V) & \forall V \in \Phi_f(\tilde{V}) \end{array} \right\}. \tag{2.1}$$

recursively for the following periods. Here,$\Phi \subseteq X = [V_0, +\infty)$ is the space of V, where V_0 is the reservation expected utility determined in the stationary equilibrium. Φ_h is the set of expected utilities that a firm can offer to a matched worker. Φ_r and Φ_f are two partition subsets of Φ with $\Phi_r \cap \Phi_f = \emptyset$. Φ_r is the retention region, and Φ_f is the termination region. In particular, if $V \in \Phi_r(\tilde{V})$, the current worker is retained; if $V \in \Phi_f(\tilde{V})$, the current worker is terminated. Here, $a : \Phi \rightarrow \Re_+$ is the recommended effort function, $c_i : \Phi \rightarrow \Re_+$ is the current consumption function conditional on realized output of θ_i, and $V_i : \Phi \rightarrow \Phi$ is the continuation value function if the realized output is θ_i. $g : G \rightarrow \Re_+$ is the termination contract the worker receives from

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the firm, where G is the space of all possible termination contracts. Note that g depends on what is available in the market and how much UI benefit a worker can claim conditional on being terminated.

Consider a firm that is matched with a worker who is indexed by $(\tilde{V}, V)$ at the beginning of a period. The firm's problem is to maximize its expected profit. Let $U(\tilde{V}, V)$ denote the value function for the firm, $U_r(\tilde{V}, V)$ denote the value associated with retention, and $U_f(\tilde{V}, V)$ denote the value associated with termination. Thus, taking the market expectation, Q, and the unemployment insurance system $B(\tilde{V}, V)$ as given, for each $(\tilde{V}, V) \in \Phi \times \Phi$ [7]

$$U_r(\tilde{V}, V) = \max_{(a, c_i, V_i)} \sum_{i=1}^{n} \pi_i(a)[\theta_i - c_i + \beta \Delta U(V, V_i) + \beta(1-\Delta)Q] - \tau, \tag{2.2}$$

subject to

$$\sum_{i=1}^{n} \pi_i(a)[u(c_i) + \beta \Delta V_i] - \phi(a) = V, \tag{2.3}$$

$$\sum_{i=1}^{n} \pi_i(a)[u(c_i) + \beta \Delta V_i] - \phi(a) \geq \sum_{i=1}^{n} \pi_i(a')[u(c_i) + \beta \Delta V_i] - \phi(a'), \forall a' \in \Re_+, \tag{2.4}$$

and

$$c_i \geq 0, V_i \in \Phi. \tag{2.5}$$

Here, equation (2.3) is the promise-keeping constraint. By definition, the allocation (a, c_i, V_i) has to deliver the expected utility V promised by the firm. Equation (2.4) is the incentive-compatible constraint. Any allocation (c_i, V_i) has to be such that it is to a worker's advantage to exert the recommended effort. Condition (2.5) is the limited liability constraint. The firm cannot make a negative compensation to the worker and the promised continuation value has to be in the space Φ. Φ will be defined in a later section. Note that the function $U_r(\tilde{V}, V)$ is indeed independent of V, and hence, $U_r(\tilde{V}, V)$ can be reduced to $U_r(V)$ for any $(\tilde{V}, V)$.

For each $(\tilde{V}, V) \in \Phi \times \Phi$,

$$U_f(\tilde{V}, V) = \max\{Q - C(g(\tilde{V}, V))\}, \tag{2.6}$$

[7]Note that there may exist some V that is not feasible for the given constraints. Let Ψ' be the set of all V that can be supported by constraints (2.3)-(2.5), and let $\Psi = \Phi \cap \Psi'$. I expand $U_r(\tilde{V}, V)$ from $\Phi \times \Psi$ to $\Phi \times \Phi$ by setting $U_r(\tilde{V}, V) = -\infty$ for all $V \in \Phi - \Psi$. The same adjustment applies to U_f.

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subject to

$$M(g(\tilde{V}, V)) = V, \tag{2.7}$$

where $C(g(.))$ denotes the cost of the termination contract $g(.)$ to the firm, and $M(g(.))$ denotes the utility entitled by termination contract $g(.)$ to the worker.

Then

$$U(\tilde{V}, V) = \max\{U_r(V), U_f(\tilde{V}, V)\}. \tag{2.8}$$

Equation (2.8) states that the firm chooses to retain or terminate the current worker by comparing the values associated with each action. The retention region and termination region are then defined by

$$\Phi_r(\tilde{V}) = \{V : U_r(V) \geq U_f(\tilde{V}, V)\} \tag{2.9}$$

and

$$\Phi_f(\tilde{V}) = \{V : U_r(V) < U_f(\tilde{V}, V)\}. \tag{2.10}$$

What is the starting expected utility a firm will offer to a matched worker? There are two types of workers in the labor market: workers never been employed and workers who are laid off. Since workers who have never been employed and new labor market entrants both have no history of outputs, they have the same reservation expected utility, V0, and hence, they can be treated the same. A firm matched with a worker V_0 decides whether or not to hire him. The value for the firm, associated with the hiring decision, is $U_r(V)$. The value associated with not hiring is the current profit 0, plus the expected profit of being in the labor market, βQ. If the firm offers a new expected utility V', it must be true that the profit of doing so is greater than or equal to the profit of not hiring. Thus, Φ_h is given by

$$\Phi_h = \{V : U_r(V) \geq \beta Q\},$$

and the optimal starting expected utility is given by

$$V^*(V_0) = \arg\max_{V \in \Phi_h, V \geq V_0} U_r(V). \tag{2.11}$$

Now consider the case in which a firm is matched with a worker who has a history of unemployment. This firm faces the same decision as the previous firm. Note that a worker who

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is laid off may receive a sequence of termination compensation from his previous employer. Similarly to Wang (2005), I assume that a firm will take away a worker's termination compensation package in exchange for a newly offered expected utility.[8] The value to a firm of recruiting an unemployed worker is the profit plus the termination compensation turned in by the matched worker. Define $\Phi_h(\tilde{V}, V)$ as the set of expected utilities that a firm will offer to an unemployed worker in state $(\tilde{V}, V)$. Then $\Phi_h(\tilde{V}, V)$ is given as

$$\Phi_h(\tilde{V}, V) = \{V' : U_r(V') + g(\tilde{V}, V) \geq \beta Q\},$$

and the optimal starting expected utility is

$$V^*(\tilde{V}, V) = \arg\max_{V' \in \Phi_h(\tilde{V}, V), V' \geq V} U_r(V'). \tag{2.12}$$

2.3.3 Government

The government collects the UI tax and uses the revenue to pay for UI benefits. Any UI policy vector (τ, b) must satisfy the condition under which the government's budget is balanced, i.e.,

$$[\varepsilon_e + (\eta - \varepsilon_e)q_f]\tau = \int_{\Lambda_a} (1 - q_w) B(\tilde{V}, V) dF_u(\tilde{V}, V), \tag{2.13}$$

where

$$B(\tilde{V}, V) = b \sum_{i=1}^{n} \pi_i(a(\tilde{V}))c_i(\tilde{V})$$

is the UI benefit that an unemployed worker claims. Note that $B(\tilde{V}, V) = B(V)$. Here, Λ_a (defined later) is the set of active workers in the labor force, $f_u(\tilde{V}, V) : \Phi \times \Phi^9 \rightarrow [0, 1]$ is the stationary PDF of active workers at the beginning of a period and $F_u(\tilde{V}, V)$ is the corresponding CDF. The left hand side of the equation (2.13) is the total UI tax revenue and the right hand side is total UI payments. ε_e is the steady state measure of workers retained at the beginning of a period. Clearly, ε_e is also the steady state measure of firms that retain their current workers. $\eta - \varepsilon_e$ is the steady state measure of firms that terminate their current workers or the number of job vacancies. q_f is the steady state probability that a firm with a

[8] By this, the game between different employers is avoided.

[9] f_u is expanded from Λ_a to $\Phi \times \Phi$ by setting $f_e(\tilde{V}, V) = 0$ for all $(\tilde{V}, V) \in \Phi \times \Phi - \Lambda_a$.

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job vacancy is matched with a worker. Hence, in the steady state, there are $\varepsilon_e + (\eta - \varepsilon_e)q_f$ number of firms operating and paying the UI tax. Next, q_w is the probability of a worker being matched with a firm. With probability $1 - q_w$, a worker who is in the labor market is not matched with a firm and, hence, claims the UI benefit.

2.3.4 Market and Equilibrium

Recall that the termination contract g depends on what is available in the economy for a worker who is terminated. Suppose the worker goes back to the labor market. Then, with probability q_w, the worker meets a firm. The firm makes its decision to hire or not hire. If the value of hiring is greater than or equal to the value of not hiring and waiting for the next period, the worker is hired. The worker is not hired otherwise. Since all firms are identical, the worker will never be employed again if one firm does not hire him. Any unemployed worker can foresee perfectly what happens in the market. In addition, I assume an unemployed worker will exit the labor market if the newly offered expected utility is lower than or equal to the current one [10] Therefore unemployed workers in the labor market are those who expect a positive probability of reemployment and whose newly offered expected utilities are greater than the ones they currently have. I call those who stay in the labor market after being terminated as involuntary layoffs and those who exit the labor market as voluntary layoffs. The termination cost $C(g(\tilde{V}, V))$ is defined, accordingly [11], as

$$C(g(\tilde{V}, V)) = \min_{c} \frac{c}{1 - \beta\Delta} \tag{2.14}$$

s.t.

$$V = \left\{ \begin{array}{ll} q_w V^*(\tilde{V}, V) + (1 - q_w)(u(B(\tilde{V}) + c) + \beta\Delta V) & \text{if } V \in \Phi_a(\tilde{V}) \\ \frac{u(c)}{1-\beta\Delta} & \text{if } V \in \Phi_{in}(\tilde{V}) \\ & \end{array} \right\} \tag{2.15}$$

[10] Note that I assume that a worker will not go back to the labor market if the offered new expected utility is equal to his current one. This assumption is actually quite reasonable. Consider the economy where workers need to make job search effort in order that the probability being matched is positive. It can be proven that any worker whose offered new expected utility is the same as the current one will devote zero effort. Those who will be offered a greater expected utility will devote positive effort. As a result, those who devote zero effort will never have the chance to be matched with a firm.

[11] I assume that a firm's termination contract is a stream of constant payments.

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Here, $\Phi_a(\tilde{V})$ is the set of agents who are to be involuntarily laid off, given their expected utilities when last employed are $\tilde{V}$:

$$\Phi_a(\tilde{V}) = \{V : U(V^*(\tilde{V}, V)) + g(\tilde{V}, V) \geq \beta Q \text{ and } V^*(\tilde{V}, V)) > V\}. \quad (2.16)$$

And $\Phi_{in}(\tilde{V})$ is the set of agents who are voluntary layoffs, given their expected utilities when last employed are $\tilde{V}$:

$$\Phi_{in}(\tilde{V}) = \{V : U(V^*(\tilde{V}, V))) + g(\tilde{V}, V) < \beta Q \text{ or } V^*(\tilde{V}, V)) \leq V\}. \quad (2.17)$$

Let ε_a be the steady state measure of involuntary layoffs at the beginning of a period, ε_{ne} be the steady state measure of workers who are never employed, and ε_{in} be the steady state measure of agents who are voluntarily laid-off. Since voluntarily laid off workers exit the labor market, the steady state measure of the labor force is $1 - varepsilon_{in}$, or $\varepsilon_e + \varepsilon_a + \varepsilon_{ne}$, and the unemployment rate at the beginning of each period is $\frac{\varepsilon_a + \varepsilon_{ne}}{\varepsilon_e + \varepsilon_a + \varepsilon_{ne}}$. Then q_w is defined as:

$$q_w = \min\{1, \max\{\frac{\eta - \varepsilon_e}{\varepsilon_a + \varepsilon_{ne}}, 0\}\}, \quad (2.18)$$

and q_f is defined by the equation below:

$$q_f = \min\{1, \max\{\frac{\varepsilon_a + \varepsilon_{ne}}{\eta - \varepsilon_e}, 0\}\}. \quad (2.19)$$

Recall that V_0 is the reservation expected utility of a new entrant. In the steady state,

$$V_0 = q_w V^*(V_0) + (1 - q_w)(u(0) + \beta \Delta V_0). \quad (2.20)$$

Q is the expected value of a firm in the labor market. As mentioned in section 2.3.2, there are two types of workers in the labor market: workers never employed, and workers who are involuntarily laid off. If a firm is matched with one of the never-employed workers, it is optimal to offer him the expected utility $V^*(V_0)$ that entitles the firm the maximum net profit $U(V^*(V_0))$. If the firm is matched with an unemployed worker $(\tilde{V}, V)$, then $V^*(\tilde{V}, V)$ is offered to the worker. The firm's expected profit is $U(V^*(\tilde{V}, V)) + g(\tilde{V}, V)$.

I define λ_{ne} as the probability of a firm being matched with an agent who has never been employed and λ_a as the probability of a firm being matched with a worker who is laid off

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involuntarily and yet active in the labor market. Then

$$Q = \lambda_{ne} U(V^*(V_0)) + \lambda_a \int_{\Lambda_a} [U(V^*(\tilde{V}, V)) + g(\tilde{V}, V)] dF_u(\tilde{V}, V) + (1 - \lambda_{ne} - \lambda_a)\beta Q, \quad (2.21)$$

where

$$\lambda_{ne} = \min\{1, \max\{\frac{q_f \varepsilon_{ne}}{\varepsilon_a + \varepsilon_{ne}}, 0\}\},$$
$$\lambda_a = \min\{1, \max\{\frac{q_f \varepsilon_a}{\varepsilon_a + \varepsilon_{ne}}, 0\}\}.$$

I define $f_e(\tilde{V}, V) : \Phi \times \Phi$ [12] $\rightarrow [0, 1]$ as the stationary probability density function (PDF) of employed workers at the beginning of a period before the labor market opens. In the stationary equilibrium, the steady state measures $\{\varepsilon_e, \varepsilon_a, \varepsilon_{in}, \varepsilon_{ne}\}$ satisfy the following rules:

$$\begin{aligned} \varepsilon_e &= \Delta\varepsilon_e \int_{\Lambda_r} \sum_{\theta_i \in \Omega_r(V)} \pi_i(V) dF_e(\tilde{V}, V) \\ &+ \Delta\varepsilon_a \int_{\Lambda_r} q_w \sum_{\theta_i \in \Omega_r(V^*(\tilde{V}, V))} \pi_i(V^*(\tilde{V}, V)) dF_e(\tilde{V}, V) \\ &+ \Delta\varepsilon_{ne} q_w \sum_{\theta_i \in \Omega_r(V^*(V_0))} \pi_i(V^*(V_0)), \end{aligned} \quad (2.22)$$

$$\begin{aligned} \varepsilon_a &= \Delta\varepsilon_e \int_{\Lambda_a} \sum_{\theta_i \in \Omega_a(V)} \pi_i(V) dF_e(\tilde{V}, V) \\ &+ \Delta\varepsilon_a \int_{\Lambda_a} q_w \sum_{\theta_i \in \Omega_a(V^*(\tilde{V}, V))} \pi_i(V^*(\tilde{V}, V)) dF_e(\tilde{V}, V) \\ &+ \Delta\varepsilon_{ne} q_w \sum_{\theta_i \in \Omega_a(V^*(V_0))} \pi_i(V^*(V_0)) \\ &+ \Delta\varepsilon_a (1 - q_w), \end{aligned} \quad (2.23)$$

$$\begin{aligned} \varepsilon_{in} &= \Delta\varepsilon_e \int_{\Lambda_{in}} \sum_{\theta_i \in \Omega_{in}(V)} \pi_i(V) dF_e(\tilde{V}, V) \\ &+ \Delta\varepsilon_a \int_{\Lambda_r} q_w \sum_{\theta_i \in \Omega_{in}(V^*(\tilde{V}, V))} \pi_i(V^*(\tilde{V}, V)) dF_e(\tilde{V}, V) \\ &+ \Delta\varepsilon_{ne} q_w \sum_{\theta_i \in \Omega_{in}(V^*(V_0))} \pi_i(V^*(V_0)) \end{aligned} \quad (2.24)$$

[12] f_e is expanded from Λ_r to $\Phi \times \Phi$ by setting $f_e(\tilde{V}, V) = 0$ for all $(\tilde{V}, V) \in \Phi \times \Phi - \Lambda_r$. Λ_r is the set of employed workers defined later.

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$$+\Delta\varepsilon_{in},$$

and

$$\varepsilon_{ne} = \frac{1-\Delta}{1-(1-q_w)\Delta} \tag{2.25}$$

Here, for all $\tilde{V} \in \Phi$, I let

$$\Omega_r(V) = \{\theta_i : V_i(\tilde{V}) \in \Phi_r(V)\},$$

$$\Omega_a(V) = \{\theta_i : V_i(\tilde{V}) \in (\Phi_f(\tilde{V}) \cap \Phi_a(\tilde{V}))\},$$

and

$$\Omega_{in}(V) = \{\theta_i : V_i(\tilde{V}) \in (\Phi_f(\tilde{V}) \cap \Phi_{in}(\tilde{V}))\}.$$

So, $\Omega_r(\tilde{V})$ is the set of realizations of θ for which the worker is retained, $\Omega_a(\tilde{V})$ is the set of all realizations of θ upon which the worker is laid off and yet active on the labor market, and $\Omega_{in}(\tilde{V})$ is the set of the realizations of θ for which the worker is voluntarily laid off.

I also let

$$\Lambda_r = \{(\tilde{V}, V) : V = V_i(\tilde{V}), \forall\, \theta_i \in \Omega_r(\tilde{V})\},$$

$$\Lambda_a = \{(\tilde{V}, V) : V = V_i(\tilde{V}), \forall\, \theta_i \in \Omega_a(\tilde{V})\},$$

and

$$\Lambda_{in} = \{(\tilde{V}, V) : V = V_i(\tilde{V}), \forall\, \theta_i \in \Omega_{in}(\tilde{V})\}.$$

Here, Λ_r is defined as the set of employed workers and Λ_{in} is the set of inactive workers.

Definition 1 *A stationary equilibrium is an optimal contract s, the expected profit of a firm in the market Q, a UI policy vector* (τ, b)*, reservation expected utility* V_0*, optimal starting expected utilities* $(V_0^*, V^*(\tilde{V}, V))$*, a job-finding rate* q_w *and a job hiring rate* q_f*, measures* $(\varepsilon_e, \varepsilon_a, \varepsilon_{in})$*, and distributions* $F(V, V)$ *where*

(i) given V_0*,Q and policy vector* (τ, b)*,* σ *solves a firm's problem 2.2;*

(ii) Q is given by (2.21);

(iii) $(V_0^*, V^*(\tilde{V}, V))$ *is defined by equations (2.11) and (2.12);*

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(iv) V_0 *is given by (2.20);*

(v) the policy vector (τ, b) *satisfies the government budget balanced condition (2.13);*

(vi) q_w, q_f *satisfy equations (2.18) and (2.19);*

(vii) $(\varepsilon_e, \varepsilon_a, \varepsilon_{in})$ *are given by equations (2.23)-(2.25);*

(viii) and finally, F *satisfies the stationarity condition:*

$$F = \Gamma(F),$$

where the operarator G *maps the distribution of the expected utilities of the employed, unemployed, and retired workers at the beginning of the current period into that of the next period, as dictated by the law of motion* (a, V_i)*, the equilibrium starting expected utility* $V_0^*, V^*(\tilde{V}, V)$*, the job-finding probability* q_w*, and the survival rate* Δ*.*

To conclude this section, this model is able to deliver the following steady state variables: (1) the measure of employment, active workers, and workers who are not in the labor force; (2) the reservation expected utility and optimal starting expected utilities; (3) the flows among employment, unemployment, and workers who are not in the labor force; (4) the job finding rate, job separation rate and unemployment rate; (5) the government budget balancing UI tax policy and UI replacement rate; (6) and finally, the optimal contract structure.

2.4 Calibration

2.4.1 Specifications

I calibrate the parameters by matching the stationary equilibrium to U.S. data. The time period in the model is a quarter. The utility function of consumption has the form

$$u(c) = \log(\alpha_0 + c),$$

where $\alpha_0 > 0$. It is clear to see that the utility function is bounded from below. The disutility of effort function is

$$\phi(a) = a^{\delta},$$

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where $\delta > 0$. The space of output $\Theta = \{\theta_1, \theta_2\}$ and the probability function associated with θ_1, θ_2 are

$$\pi_1(a) = e^{-\gamma a} \text{ and } \pi_2(a) = 1 - e^{-\gamma a},$$

where $\gamma > 0$.

There are 9 parameter values to set in order to compute a stationary equilibrium. They are values for the output θ_1 and θ_2, the discount factor β, the survival rate Δ, the lower bound of the utility function $\log(\alpha_0)$, the parameter in the disutility-of-effort function δ, coefficient of effort effectiveness γ, UI benefit replacement ratio b, and number of firms η. Low output θ_1, is set to 0. $\beta = 0.99$, corresponding to an annual interest rate of 4.93%. Δ is set to 0.9950, which implies a working life of 50 years. δ is set to 2. Replacement ratio b is set to 0.5, in line with most works and consistent with what is in practice in the U.S.

The remaining four parameters are α_0,θ_2, γ, and η. I set $\alpha_0 = 0.5$. Ideally, I would like to compute η as the ratio of the addition of job openings and employment to the number of workers in the economy. However, the data on job openings are unreliable, which implies that η has to be calibrated. Hence, θ_2, γ, and η are set to replicate the observed measures of employed workers, actively unemployed workers, and workers not-in-the-labor-force in the steady state. Blanchard and Diamond (1990) study a sample period running from January 1968 to May 1986. They derive steady state numbers of employment, unemployment, and of the not-in-the-labor-force. I convert these monthly stock numbers into quarterly numbers and derive the corresponding measures. For that sample period, measures of employment, unemployment, and workers who are not in the labor force are 0.5936, 0.0414, and 0.3650 respectively.

2.4.2 Computation Method

Equilibrium is computed using the iteration method. In more detail, I first guess a firm's expectations of the market: reservation expected utility V_0, and Q. Taking those as given, I then guess the functional forms of U and g. Next, for a given $\{V_0, Q, U, g\}$, I guess the budget-balancing tax τ. Next, optimal effort levels are computed for each state, and a new

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value function U is derived. I then iterate on U again until the convergence criteria for U is satisfied. That is, a steady state function U consistent with the given g and Q is found. Sets $\{\Phi_r, \Phi_f, \Phi_a, \Phi_{in}\}$ are derived by comparing U_r with U_f, and an optimal rule of starting expected utilities $V^*(\tilde{V}, V)$ is also computed. Those sets and optimal effort levels are used to compute a steady state distribution of agents across different employment statuses and expected utilities. Steady state measures are then computed. Furthermore, total UI payments and total UI tax revenue are also derived. To balance the government budget, a new UI tax is computed. Once the government budget balance condition is satisfied, I then proceed to update Q, g and V_0. The computation steps listed above repeat until the convergence criteria for set Φ and functions Q, g and U are reached.

2.4.3 Features of the Calibrated Solution

For $\theta_2 = 4.465$, $\gamma = 4.328$, and $\eta = 0.6054$, the steady state measure of active unemployment is 0.0411, for employment is 0.5942, and for retired workers, 0.3647. These statistics closely match the stock features of the data set studies by Blanchard and Diamond.

It is useful to study the characteristics of the calibrated solution. Fig. 2.1 displays a firm's value functions. Fig. 2.2 shows the optimal effort recommended by a firm and Fig. 2.3 displays the stationary distribution of employed workers.

In Fig 2.1, the solid line is the value associated with retention, U_r. The dotted line is the value associated with termination, U_f. Termination cost is shown by the dashed line. The first two lines cross each other on both ends, which implies that termination can happen under two situations. First, if a worker experiences a sequence of low outputs, he will be terminated. This is because the firm loses by retaining him. The second case is, if a worker experiences a sequence of high outputs, he will also be terminated. Two thresholds of the expected utilities (V_a, V_{in}) exist in the labor market. Any worker whose expected utility is lower than V_a will be laid off and stay in the labor market; any with an expected utility higher than V_{in} will be laid off and exit the labor market. There is a sudden rise on the left end of the dashed line, which reflects that most of the termination cost has been paid by the government and, hence,

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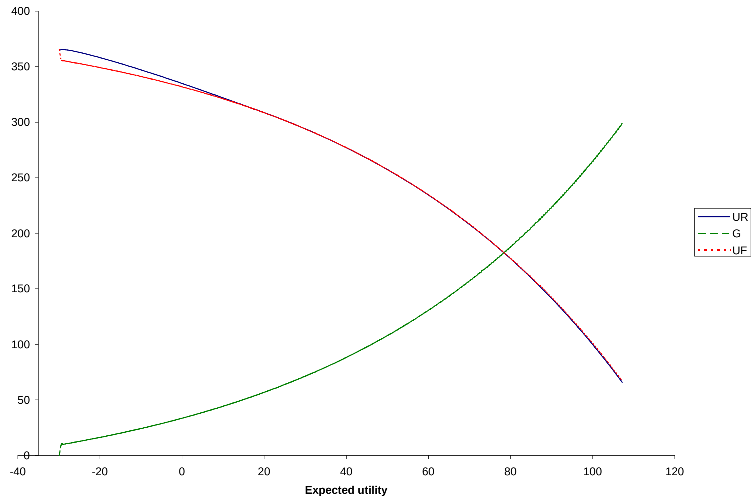

Figure 2.1 Optimal value functions

the firm only pays a small amount.

With few exceptions, the optimal effort level (a) increases first, then starts to fall as expected utility increases. In other words, a poor (low V) worker or a rich (high V) worker exerts low effort. This is the result of two constraints: the incentive scheme and the promise-keeping constraint. I first explain the incentive scheme. To induce high effort for a given expected utility, a firm will increase the difference in a worker's payoff between high output and low output. The greater the difference, the higher the effort a worker exerts. That is, a worker will be punished if low output occurs and rewarded if high output occurs. Next, the promise-keeping constraint limits the possible punishments and rewards. Consider a poor worker. A firm can only punish him to a certain extent because of the non-negativity consumption constraint. A poor worker can not be reward too much either because of the promise-keeping constraint. Therefore, a poor worker makes little effort. Consider a rich worker. A firm can punish him, but rewarding him becomes harder since the marginal utility is quite low for a rich worker. The promise-keeping constraint then places a lower bound for the punishment and, hence, a rich worker also makes little effort. A poor worker makes low level of effort because the firm cannot punish him very much; a rich worker makes low level of effort because he is too hard to motivate.

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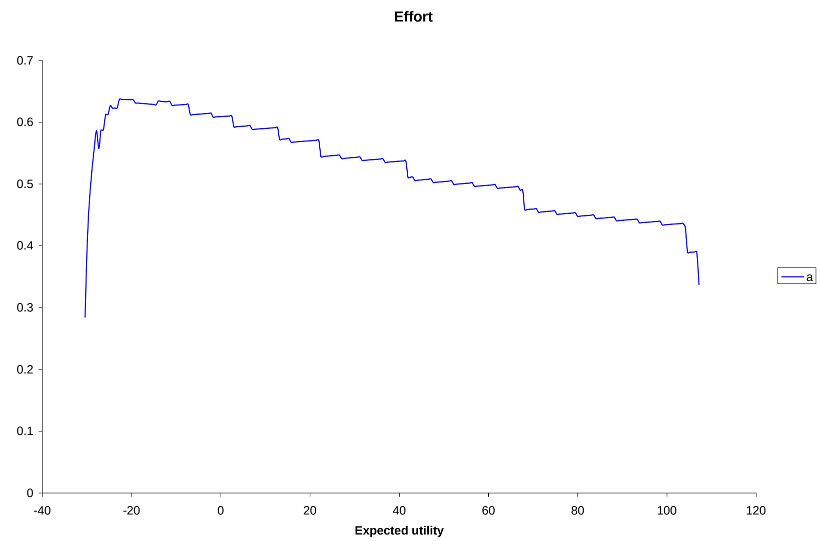

Figure 2.2 Optimal effort functions

It is worthwhile to point out that on the left end of the graph, there exists a point at which the optimal effort suddenly drops. The optimal effort function appears steeper before that point. This is not surprising because the firm now has one more scheme to motivate effort: termination. Imagine an economy where termination is not allowed. A firm can promise a very low expected utility to induce a higher effort so that current profit is maximized. Nevertheless, the firm's future profit is low because a worker will make little effort given that he is promised a low expected utility. There exists a trade-off between today's profit and tomorrow's profit, and the optimal effort may not be the highest effort feasible. Now imagine an economy where termination is allowed. A firm now can motivate the highest effort without undermining its future profit. This is because if low output occurs, the worker will be terminated. The firm can now go back to the labor market to obtain a higher profit. Hence, when termination is allowed, firms can motivate greater effort. As a worker's expected utility increases, the firm relies less on termination and more on punishment and rewards. At some point, the firm will use the latter two only. This causes the optimal effort to decrease. If the same logic applies to rich agents, there will exist a point at which the effort level rises. This does not happen for several reasons. First, rewarding a rich agent with consumption would imply a greater cost relative to

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a poor agent, because the marginal utility of the rich agent is lower. Second, rewarding a rich agent with a high continuation value implies a higher termination cost. Hence, inducing a high level of effort, either by incentive scheme or termination scheme, does not seem so appealing.

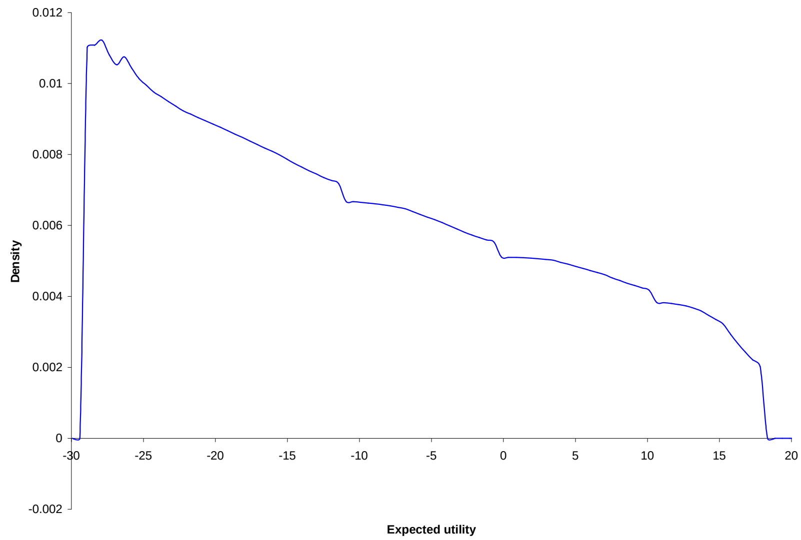

Figure 2.3 Distribution of expected utilities, employed workers

Since most of the population is employed, the distribution is dominated by the behavior of employed agents. Fig. 2.3 shows the steady state distribution of expected utilities across employed workers. The fraction of agents is a decreasing function of th expected utility. A small fraction of agents hold high expected utilities, while a large fraction of agents are around the left end. Distributions are degenerate for unemployed yet active workers and workers not-in-the-labor-force [13] hence they are not displayed here.

The stationary unemployment rate is 6.47%. With a probability of 0.275, an unemployed worker becomes employed. About 10.4% of the workers are separated from their previous jobs. Among them, 77.6% of the workers flow into unemployment, and 22.4% of the workers exit the labor market. Given that the number of firms is fixed, and it is always less than the number of the workforce, any firm with a vacancy is matched to an active agent with probability 1.

[13] This is due to the number of grids used in the computation. With more grids, the computation is able to generate two non-degenerate distributions. But the tradeoff is a high computation burden. I now ignore this issue.

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2.5 Numerical Experiments

2.5.1 Replacement Rate

I first examine the effects of removing unemployment insurance by setting the replacement ratio $b = 0$, given the same parameter settings as outlined above. Table 2.1 compares the firm's termination decisions with no UI and under the existing system with $b = 0.5$. The threshold V_{in} decreases from 18.86 to 18.34. It suggests that a worker will be terminated in an earlier period of his contract with a lower expected utility with the existing UI system. Threshold V_a is not changed, which implies that a firm's termination decision under the low output state is unaffected.

Table 2.1 Firm's Termination Decision

	V_a	V_{in}
No UI	-29.93	18.86
Existing UI	-29.93	18.34

Table 2.2 Labor Force Components

	ε_{ne}	ε_a	ε_r	ε_{in}
No UI	0.0206	0.0259	0.5943	0.3593
Existing UI	0.0179	0.0232	0.5942	0.3647

Table 2.2 gives the steady state measures of workers with different statuses. In contrast to the economy without a UI system, there are more workers who are not in the labor force, fewer workers unemployed, and less employment. The first result is due to earlier termination. A worker now faces a higher probability of being fired conditional on high outputs. Hence, more retirements appear. This also implies fewer workers in the labor force and more job vacancies. With a smaller labor force and more job vacancies, a worker in the market now faces a higher probability of being hired. Table 2.3 compares the steady state labor turnover rates. j_a measures the ratio of the number of agents who are laid off, and yet active, to the number of employed. j_{in} measures the ratio of the number of agents who are laid off and exit the labor market to the number of employed. q_w is the probability of an active agent

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being matched to a firm, and UR is the unemployment rate. As explained above, q_w increases from 0.2387 to 0.2725, while the unemployment rate is reduced from 7.26% to 6.4%. Note that not only is there more separation for rich workers, more separation for poor workers also arises. This is because more poor workers flow into employment. With more poor workers, probabilities of low output increase and job separation increases.

The first 2 columns of Table 2.4 compare the average expected utility and average effort for two environments. Contrary to what has been delivered in the literature, a UI system decreases the average expected utility, although the change is very small. This is because agents leave the labor market with less wealth. The average effort rises rather than falling. This is also caused by earlier termination. Since those who are terminated exert low levels of effort, it is obvious that terminating those agents pushes the average effort up. The last column gives the welfare loss relative to the baseline. It is computed as the percentage of consumption which, when subtracted from the consumption of each agent across different employment status, equates the mean expected utility in the case with no UI to mean expected utility with UI. Despite all of these changes of the average variables, the welfare loss is negligible.

Table 2.3 Labor Turnover Rate

	j_a	j_{in}	q_w	UR
No UI	0.0104	0.003	0.2387	7.26
Existing UI	0.0105	0.0031	0.2725	6.47

Table 2.4 Aggregate Variables

	Mean a	Mean V	Mean output	Welfare gain	Tax (%)
No UI	0.3655	-0.714	2.5013	0.00%	0.00
Existing UI	0.3657	-0.719	2.5015	-0.033%	0.037

Now I examine the effects of a UI system on firms' termination behaviors, labor market transitions and welfare change by varying the replacement rate. Table 2.5 gives the cutoff points V_a, V_{in}, and V^*. Again, if a worker's expected utility falls below V_a, he will be terminated conditional on low output; if a worker's expected utility is above V_{in}, he will be terminated when high output is realized. V_{in} is not affected when the replacement rate is low. As replacement

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increases, V_{in} starts to decrease, implying termination in an earlier period. V_a is the same [14] for all of the given replacement rates. Given the number of grids used in the computation, this result does not suggest that termination behavior will never be affected by replacement rate. A UI system affects the economy by several means. First, it alleviates a firm's termination cost and increases its profit from going back to the labor market. Therefore, termination is more likely to happen. Second, a firm's current profit of retention is reduced due to a UI tax, which further reduces a firm's expected profit from going back to the labor market. This will make the firm hesitate to terminate the current worker and go back to the labor market for other opportunities. For the environment defined by calibrated parameters, when the replacement rate is low, these two effects act against each other, resulting in no change in a firm's termination behavior. As the replacement rate increases, the benefit of a lower termination cost is greater than the cost of a lower expected profit, causing a lower V_{in}.

Table 2.5 Firm's Termination Decision

Replacement rate	V_a	V_{in}
0.1	-29.93	18.86
0.3	-29.93	18.34
0.5	-29.93	18.34

Table 2.6 Labor Force Components

Replacement rate	ε_{ne}	ε_a	ε_r	ε_{in}
0.1	0.0204	0.0256	0.5942	0.3597
0.3	0.0179	0.0227	0.5942	0.3652
0.5	0.0179	0.0232	0.5942	0.3647

Table 2.6 compares the steady state measures for different replacement rates. It seems that the current environment reacts to the replacement rate in a slow-moving manner. Solutions for the replacement rate $b = 0.0$, 0.1 are the same, while solutions for $b = 0.3$, 0.5 are almost identical. Once again, the number of grids may contribute to this sluggish response. I now only compare the cases where the replacement rate is 0.1 and 0.3. Consistent with the changes

[14] Here, $V_{\max}$ and V_{in} are unaffected by the replacement rate, mainly due to the relatively few number of grids used in the computation process. For a large number of grids, V_a, $V_{\max}$ and V_{in} will be affected by the replacement rate.

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in V_{in}, a higher replacement rate induces more voluntary layoffs in the economy. The measure of employment is reduced slightly in the steady state. Unemployment is brought down as the replacement rate goes up. There are fewer agents who are never employed and fewer agents who are involuntarily laid off in the stationary equilibrium. Table 7 gives the labor turnover rates. The changes in labor turnover rates can be ignored.

Table 2.7 Aggregate Variables

Replacement rate	Mean U	Mean a	Mean c	Welfare Gain	Tax (%)
0.1	-0.7021	0.3654	0.6002	0.026%	0.009
0.3	-0.7112	0.3657	0.6003	0.00%	0.022
0.5	-0.7210	0.3656	0.6006	-0.033%	0.037

Table 2.7 gives the steady state average variables of the economy for different replacement rates. The highest average expected utility is achieved with the replacement rate of 0.1 with a welfare gain of 0.026%. However, this welfare gain is also too small. As the replacement rate rises, the average expected utility starts to decrease; nevertheless, the average effort and output increases. The average consumption increases, as well. As mentioned above, an unemployment insurance system affects a firm's termination behavior by two means: taxes and unemployment benefits. With a high replacement rate, it is more likely that the second effect dominates the first effect. In other words, the magnitude of the decrease in a firm's expected profit is greater than the cuts in termination costs. As a result, a firm terminates a worker in an earlier period of the ongoing contract with a lower expected utility. That furthers reduces the average expected utility. The welfare loss is 0.033%, which is negligible.

I now restate the effects of the replacement rate. For the given environment, when a replacement rate is sufficiently high, firms' termination decisions are affected in that workers are more likely to be terminated at an earlier period in their contracts. The job separation rate rises, with a large fraction of workers flowing from employment into voluntary layoffs. The job finding rate increases because there are more vacancies, but fewer available workers in the labor market. Unemployment rate is reduced as the result of a smaller labor force. The average expected utility goes down, while the average effort, output, and consumption increase. The

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welfare loss induced by the replacement rate is negligible.

2.6 Conclusion

This paper is a quantitative study of the effect of a UI system on labor market outcomes from a firm's perspective in a fully dynamic contract model with endogenous layoffs. The results of the paper suggest that a UI system induces firms to terminate workers more often than it would in its absence. Numerical computations also indicate that the UI system may reduce worker welfare. The model suggests that firms offering dynamic employment contracts may be able to do a better job of insuring workers against layoff risk without much need for a publicly provided UI system.

This study contributes to the ongoing research on the UI system in several dimensions. First, it isolates the effect of a UI system on firm behavior. Second, the welfare analysis takes voluntary layoffs into account and calls into question existing work on optimal design of UI systems that models layoffs as an exogenous process.

It is important to note that my paper does not negate the important role played the government in insurance provision to the unemployed. As caveats, one must point out that my work ignores the effect of the UI system on a worker's search intensity. It also ignores issues such as possible lack of commitment by firms to pay severance payments, saving by workers, and so on. Incorporating some of these issues into the current setup would no doubt add more realism to the structure.

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CHAPTER 3. UNEMPLOYMENT INSURANCE WITH BORROWING AND UITF

3.1 Abstract

This paper evaluates the effects of the current unemployment insurance (UI) program in a model where agents are able to borrow and lend, and the government holds an unemployment insurance trust fund (UITF) balance. The only borrowing constraint in this economy is that an agent's consumption must be non-negative. It appears that the current positive UITF balance improves the welfare by 0.65%, relative to the economy with a UITF balance of 0 and yet the existing UI program is still available. In addition, removing the current UI program from the economy improves the welfare by 1.79%, which suggests that the public UI program may not be needed when agents are able to smooth consumptions by borrowing.

3.2 Introduction

Over the last 3 decades, there have been a large amount of research that study the effects of the unemployment insurance (UI) program. On one hand, the UI program provides temporary financial assistance to unemployed workers, which helps them smooth their consumptions. On the other hand, the opportunity cost of being unemployed is reduced by the unemployment benefit. Hence, the UI program may induce a higher unemployment rate since workers tend to exert lower job searching effort. The evaluation of the UI program therefore requires a careful investigation for the magnitude of both the benefits and the costs. However, as is well known, individuals are able to self-insure themselves against the unemployment risk by using savings or borrowing to finance their consumptions. And this may affect the magnitude of the effects

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of the UI program. The better the self-insurance scheme is, the less the benefit and the larger the cost that a UI program may generate. Nevertheless, in the literature, the availability of the self-insurance scheme is more or less limited. For instance, Shavell and Weiss (1979), Wang and Williamson (1996) and Hopenhayn and Nicolini (1997) preclude borrowing and lending, while Hansen and Imrohoroglu (1992), Wang and Williamson (2002) allow agents to save in the form of non-interest-bearing assets. Eric Young (2004) allows agents to save by an endogenously determined interest rate, however, agents are unable to borrow. This paper differs itself from the previous studies by considering a better[1] self-insurance scheme. That is, agents are able to borrow and lend, with the constraint being that consumption must be non-negative.[2]

A second feature that makes this paper distinct from the rest is that it includes the unemployment insurance trust fund (UITF). In U.S., UI is a federal-state joint program. Every state's unemployment insurance program has a trust fund that contains all employer contributions (payroll taxes) and benefit reimbursement payments (paid by non-profits and governmental employers in lieu of payroll taxes) from that state. Each UI trust fund is an interest-bearing account in the U.S. treasury. And state and federal law provides that UI trust funds are dedicated exclusively to the payment of UI benefits. Data suggests that many states hold positive UITF balances most of the time. This could possibly affect the economy in several ways. First, a positive UITF balance can be viewed as forced savings, which may crowd in or out the private savings. This implies that both the equilibrium UI tax rate and interest rate may be altered. Therefore, the government is able to use the UITF balance to affect individuals behavior through the channel of equilibrium rates. Second, a positive UITF balance can be used to pay for the increases in the UI benefit claims during recession periods, which helps to relieve part of the tax burden imposed on employed workers. This paper focus on the former effect while abstracts away from the latter one, since I concentrate on a stationary equilibrium where cycles do not exist.

[1] In this paper, agents are able to borrow with no limits, while much of the literature have assumed an ad-hoc borrowing constraint or an endogenous one. The latter one would imply agents cannot borrow when they are unemployed. However, in nowdays, the popularity of credit card uses has relaxed this constriant in some extent, which makes this assumption reasonable somehow.

[2] Eric Young (2004) also considers a similar self-insurance scheme in one of the sections. However, agents face an endogenous borrowing constraint, which does not exist in my paper.

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This model is built along the line of Wang and Williamson (2002). The economy is populated by a continuum number of infinitely-lived agents. There is a production technology and a capital market in the economy. Agents are born identical. However, as time goes by, they can be different across employment status and asset/debt holdings. At the beginning of each period, an agent makes effort and gains an access to the production technology with a probability. This agent is considered being employed. The probability of being employed is a function of an agent's effort level. If an agent is employed, he produces y units of consumption good. If not employed, he produces 0. Once their income are determined, agents make saving or borrowing decisions for the next period. They then consume the rest of the income and proceed to the next period. There is also a government in the economy. The government runs a unemployment insurance trust fund program, collecting tax revenues from employed workers and making payment to unemployed agents eligible for UI benefits. The government does not necessarily hold a zero-balanced budget.

I calibrate the model to replicate the U.S. labor market for the period 2005 and 2006. I then conduct two policy experiments. The first experiment is to set the UITF balance to zero, i.e., a zero balanced government budget. The second is to consider the case when the public UI program is not available. I then compare the stationary equilibrium solutions of the baseline model with those of the two experiments. There are two important findings. First, the current UITF balance improves the welfare by 0.65%, relative to the economy where the UITF balance is zero and yet the public UI program is still available. Next, removing the current UI system improves the welfare by 1.79%, higher than what have been found in most of the literature. Other findings are: (1) the current UI program reduces the effort level by at most 8% for employed agents, 46% for unemployed agents receiving UI and almost 0% for those not receiving UI benefits. (2) the UI system has very little effects on employed agents' saving and borrowing decisions. However, agents receiving UI benefits experience a large increase (decrease) in their asset (debt) holdings, ranging from 28% to 49%. (3) The fall in consumption upon unemployment is reduced by at large 66% due to the unemployment insurance benefits, but only 12% for those who stay longer in the unemployment spells. Similar

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to what has been shown in Gruber (1997), UI serves as a short-run consumption smoothing device. However, the magnitude of the effect is much larger. (4) At the aggregate level, the current UI including the positive UITF balance increases the unemployment rate from 4.2% to 4.8%. (5) The current UI and UITF together crowd out the total savings by 6.87% and the private savings by 20.81%. UI alone reduces the total savings only by 1.78%. In a word, a positive UITF balance can be beneficial to the economy when the UI program is present, and the distortionary effects of UI is larger for a better self-insurance scheme.

This paper is along in line with many work in the UI analysis. Gruber (1997) and Engen and Gruber (2001) present some empirical evidences. Gruber (1997) assesses the benefits of UI by measuring the effect of this program on consumption smoothing during periods of joblessness. He uses regression methods to estimate the relationship between consumption changes upon unemployment and after unemployment spells and the level of UI benefits. In addition, Gruber evaluates the optimal benefit level by applying Baily's model (1978). His finding is that the optimal UI benefit level should be zero when the degree of risk aversion is lower than 2. My result is consistent with this finding. For the case when the coefficient of risk aversion is 1, an economy without the UI program actually improves the welfare. Engen and Gruber (2001) addresses the question that if UI shall have a noticeable effect on individual's precautionary saving behavior. They find that the current UI system can lead to a significant reduction in the assets accumulated by a median worker. Moreover, reducing the UI benefit replacement rate would increase aggregate asset holdings. In my model, the current UI only reduces the total savings by a small percentage.

More recent papers focus on the optimal UI design problems. Shavell and Weiss (1979), Hopenhayn and Nicolini (1997) are among the first theoretical papers that investigate the optimal unemployment insurance design in a setting that precludes savings. Hansen and Imrohoroglu (1992) employ a quantitative dynamic general equilibrium model to study the same problem where agents can only save in a non-interest-bearing form. Costain (1997) combines moral hazard problem and precautionary savings to analyze quantitatively the costs and benefits of UI. He found that even without UI, workers smooth consumption effectively through

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asset accumulation. And the welfare gain from UI is rather small. Wang and Williamson (2002) also allow for savings in the incomplete market economy and derive the same welfare result as Costain's. Eric Young (2004) builds on the model by Wang and Williamson and adds in capital input to the production technology. Aggregate savings equal the demand for capital in the equilibrium, and hence, interest rate is endogenously determined. His findings are that the UI program causes a relatively large decline in aggregate activity. Consequently, the optimal replacement rates are always zero. It is worthy to note that Young also considers the case when agents are able to borrow, but subject to an endogenous borrowing constraint. His finding is that the removing UI improves the welfare. My results support Eric Young's finding, but the magnitude is slightly larger. However, it does not suggest the other findings are wrong since we focus on different extent of the self-insurance scheme. However, it suggests that the extent of the self-insurance scheme may significantly affect the magniture of the UI effects.

It is probably safe to say that my model is more closely related to Wang and Williamson (2002). What differs my paper is that agents are allowed to borrow any amount of consumption goods as long as their consumptions are non-negative. This will relax some of the precautionary saving motives. Agents do not need to save as much as before when borrowing is not allowed for their consumption during bad times. Furthermore, the existence of a positive UITF balance can be viewed as a force public saving, which affects the interest rate, and hence has some impacts on agents' consumption-smoothing and job-searching effort behavior.

The rest of the paper is organized as follows. Section 2 describes the model economy and defines the stationary equilibrium. Section 3 calibrates the model to U.S. data and analyzes the calibrated solution. Section 4 presents the results for two policy experiments. Section 5 then concludes.

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3.3 Model

3.3.1 Environment

There are a continuum of infinitely-lived agents. An agent's preference can be represented by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(a_t)], \tag{3.1}$$

where β is the discount factor, and c_t, a_t denote the levels of consumption and effort at period t. $u(c_t)$ is the period utility of consumption and $v(a_t)$ is the period disutility of exerting effort. I assume that $u(\cdot)$ and $v(\cdot)$ are concave and convex functions respectively, and they are both twice differentiable. There is a riskless project in the economy. The only input in this project is one agent's inelastic labor and the output is y units of consumption good. We consider agents who have access to the project as being employed. The number of the project can be unlimited. However, the probability of gaining the access to the project is depending upon one's effort level and employment status. Here, we let $\gamma_s(a)$ denote the probability of being employed (or taking the project) for an agent who is in UI status s and exerts an effort level of a. If an agent does not have access to the project, or if he is unemployed, he receives no consumption good for the current period.

There is only one consumption good in the economy. I assume that 1 unit of the consumption good can be easily transformed to 1 unit of capital good without incurring any cost. Each agent is allowed to borrow or lend his/her assets in the credit market. If one puts 1 unit of capital good in the credit market at period t, he/she can receive R units of capital goods at the beginning of next period.

There is a government in the economy. The government runs a UI program and operates a Unemployment Insurance Trust Fund (UITF) account. Each period, the government collects payroll taxes t from employed workers and puts it in the UITF account. The government then makes payment, b, to each unemployed agent eligible for receiving UI benefits. If the tax revenue exceeds the total unemployment benefit payments, the remainder is invested in the credit market. Similarly, if the tax revenue is not sufficient enough for the payments,

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the government borrows the difference from the credit market. I define B_t as the difference between the tax revenue and UI payments at the end of period t. B_t can be positive, negative and zero.

Whether or not an unemployed agent is eligible for receiving UI benefit, depends on his employment status. If this agent was employed for the previous period and becomes unemployed for the current period, then with a probability of θ, he receives UI benefit for the current unemployment spell; with a probability of $1-\theta$, he is not eligible for receiving benefit for the current unemployment spell, but maybe eligible for future unemployment payment. If one receives UI benefit, he/she can receive it for m consecutive unemployment periods.

3.3.2 Individual's Problem

I let $V(s,k)$ denote the value function for an agent who is in "UI status" s and has saved k units of assets (capital goods) during last period. Here, $s=(j,l)$, where j denotes the agent's employment status at the beginning of the period, with $j=0,1,2,...$, and l denotes the eligibility state, i.e. $l=1$ if the agent is eligible for receive UI benefit, and $l=0$ otherwise. Furthermore, $j=0$ denotes an agent who is employed during the last period, $j=k$ denotes an agent who has had k consecutive periods of unemployment. Hence, $\gamma_s(a)$, the probability of being employed, now can be rewritten as $\gamma_j(a)$.[3] Then the Bellman equations associated with the agent's dynamic optimization problem are given by

$$\begin{aligned} V((0,1),k) &= \max_{\{a,k_0,k_1,k_2\}} \gamma_0(a)[u(kR+y-\tau-k_0)+\beta V((0,1),k_{0)}] \\ &\quad +(1-\gamma_0(a))[\theta(u(kR+b-k_1)+\beta V((1,1),k_1)) \\ &\quad +(1-\theta)(u(kR-k_2)+\beta V((1,0),k_2))] \\ &\quad -\upsilon(a) \end{aligned} \tag{3.2}$$

subject to

$$kR+y-\tau-k_0 = 0, \tag{3.3}$$

[3]Note that the notation, $\gamma_j(.)$, implies the probability of being employed does not depend on eligibility status. The probability of being employed is more related to the quality of the human capital an agent carries. As an agent stays longer being unemployed, his human capital may depreciate. However, receiving UI benefits or not almost has no effect on the quality of human capital.

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$$kR + b - k_1 = 0, \quad (3.4)$$

$$kR - k_2 = 0, \quad (3.5)$$

$$\begin{aligned} V((j,1),k) = & \max_{\{a,k_0,k_1\}} \gamma_j(a)[u(kR + y - \tau - k_0) + \beta V((0,1),k_0)] \quad (3.6) \\ & +(1-\gamma_j(a))[u(kR + b - k_1) + \beta V((j+1,1),k_1)] \\ & -\upsilon(a) \end{aligned}$$

for $0 < j < m-1$, subject to

$$kR + y - \tau - k_0 = 0, \quad (3.7)$$

$$kR + b - k_1 = 0, \quad (3.8)$$

$$\begin{aligned} V((j,1),k) = & \max_{\{a,k_0,k1\}} \gamma_j(a)[u(kR + y - \tau - k0) + \beta V((0,1),k_0)] \quad (3.9) \\ & +(1-\gamma_j(a))[u(kR + b - k_1) + \beta V((j+1,0),k_1)] \\ & -\upsilon(a) \end{aligned}$$

for $j = m-1$, subject to

$$kR + y - \tau - k_0 = 0, \quad (3.10)$$

$$kR + b - k_1 = 0, \quad (3.11)$$

$$\begin{aligned} V((j,0),k) = & \max_{\{a,k_0,k_2\}} \gamma_j(a)[u(kR + y - \tau - k_0) + \beta V((0,1),k_0)] \quad (3.12) \\ & +(1-\gamma_j(a))[u(kR - k_2) + \beta V((j+1,0),k_2)] \\ & -\upsilon(a) \end{aligned}$$

subject to

$$kR + y - \tau - k_0 = 0, \quad (3.13)$$

$$kR - k_2 = 0. \quad (3.14)$$

Here, k_0 denotes the quantity of assets held by an agent if he is employed this period, k_1 denotes the level of the end-of-period asset if he is unemployed and receives benefit this period,

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and k_2 denotes the end-of-period assets if he is unemployed and does not receive benefit. Note here that k_0, k_1, k_2 can be negative, which mean debts.

Equation (3.3) represents the maximum lifetime utility an agent employed last period is able to achieve, by optimally choosing his effort level a, assets/debts holdings k_0, k_1, k_2 for next period and current consumption. His income for the current period is $y - \tau + kR$ if he stays employed, $b + kR$ if he becomes unemployed and yet receiving UI benefits and kR otherwise. Equations (3.7)-(3.13) represent the same optimal problems, but for unemployed agents with different employment status. The only constraint here is that consumption cannot be negative.

3.3.3 Market

Each period, agents behave by their optimal decision rules $\{a_s(k), k_0(s,k), k_1(s,k), k_2(s,k)\}$. Individuals' optimal behavior therefore governs the distribution of all agents across the employment status and the level of asset/debt holdings. We define $\mu^t_{j,l}(k)$ as the fraction of agents whose employment status are (j,l) and whose asset/debt holdings are k at the beginning of period t. Then it must be the case that $\sum_{j,l,k} \mu^t_{j,l}(k) = 1$.

Once the distribution of agents is determined, we are ready to define the government's budget balance. During period t, the government's total revenue is the total tax collected from all employed workers and the interest earnings of the UITF balance accumulated from period $t-1$, that is, $B_{t-1}R$. The government uses the total revenue to make payments to unemployed agents who are eligible for UI benefits. The government then invests the remainder in the credit market. If the revenue is not large enough to pay for the UI benefits, the government borrows the difference from the credit market. Hence, the government intertemporal budget constraint is

$$\begin{aligned}\sum_{j,l,k} \mu^t_{j,l}(k)\gamma_j(a^t_{j,l}(k))t + B_{t-1}R &= \theta \sum_k \mu^t_0(k)(1-\gamma_0(a0^t(k)))b + \\ &\sum_{1\le j\le m-1,k} \mu^t_{j,1}(k)(1-\gamma_j(a^t_{j,1}(k)))b + B_t,\end{aligned} \tag{3.15}$$

where the first term of the LHS of the equation is the total tax revenue. The first term of the

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RHS is the UI payments to the agents who just become unemployed. They claim UI benefits with a probability θ. The second term denotes the payments to the agents who receive benefits for the later periods of their unemployment spell. B_t denotes the new balance for period t.

We next define the credit market clearing condition. Total supply of capital goods includes the private savings of individuals, and the public savings of government. For an agent who holds k units of capital at the beginning of period t, he will accumulate asset k_0, k_1, k_2 at the end of period t with corresponding probabilities. Hence, the total private savings at period t is the addition of savings of all individuals at the end of period t. At the beginning of period t, government holds a trust fund balance of $B_{t-1}R$. After using tax revenues and interest earnings to finance the unemployment benefits, government has a new trust fund balance B_t. Therefore, the credit market's clearing condition is

$$\sum_k \mu_0^t(k)[\gamma_0(a_0^t(k))k_0((0,1),k) + (1-\gamma_0(a_0^t(k)))(\theta k_1((0,1),k) + (1-\theta)k_2((0,1),k))] \quad (3.16)$$
$$+ \sum_{1\le j\le m-1,k} \mu_{j,1}^t(k)[\gamma_j(a_{j,1}^t(k))k_0((j,1),k) + (1-\gamma_j(a_{j,1}^t(k)))k_1((j,1),k)]$$
$$+ \sum_{j,k} \mu_{j,0}^t(k)[\gamma_j(a_{j,0}^t(k))k_0((j,0),k) + (1-\gamma_j(a_{j,0}^t(k)))k_2((j,0),k)] + B_t = 0$$

Note that the eq. (3.17) incorporates the case when assets and trust fund balance are negative, i.e., debts.

The first term in the eq. (3.17) is the total savings/debts accumulated by agents who are employed in period $t-1$. The second term denotes the total savings/debts held by agents who are unemployed last period and receive UI benefit. The third terms is the assets of agents who are unemployed last period and yet not receiving UI benefits. The last term is the government's public savings/debts.

We are now ready to define the steady state equilibrium.

Definition 1 *A steady state equilibrium is given by a stationary distribution μ, a UI policy vector τ, b, a UITF balance B and an interest rate R such that*

(i) $\Gamma\mu = \mu$,

(ii) the government budget constraint, i.e., the equation (3.16) holds,

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(iii) the credit market clears, that is, the equation (3.17) holds, with the mapping Γ *derived from the optimal decision rules which are the solution to the problems (3.3)-(3.13).*

In summary, this model is able to generate the profile of the labor market force, the unemployment rate, the component of total private savings and the government UITF balance, in an environment where a public UI is available and agents are allowed to borrow and lend. Following the standard literature, in the next section, I calibrate the model parameters by matching the steady state equilibrium outcomes to the U.S. labor market and use that to study individuals' optimal behavior and aggregate market performance.

3.4 Calibration

3.4.1 Calibrated Parameters

I calibrate the model to replicate the U.S. labor market quartly data for the period 2005 and 2006. I adopt the forms of utility function, disutility function of effort, probability of being employed assumed by Wang and Williamson (2002). The utility function is

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$$

where σ is the coefficient of relative risk aversion, with $s > 0$. The disutility function of effort is

$$v(a) = a^{\delta}$$

where $\delta > 0$. The function determining the probability of employment is

$$\gamma_j(a) = 1 - \exp^{-r_j a}$$

where $r_j > 0$ for all j. Here, we also set $m = 2$ [4] , so that there are four unemployment status: (0,1), (1,0), (1,1) and (2,0), where (0,1) denotes the agents who are employed for the last period and henceforth eligible for receiving UI benefit if unemployed for the current period, (1,0) represents those agents who have been unemployed for one period and not eligible for

[4] In most states, unemployed agents can only receive UI benefits up to 26 weeks, which is almost equivalent to 2 quarters, i.e., 2 periods in this paper.

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receiving UI benefit. If one has been unemployed for one period, but eligible for receiving UI benefit, his employment status is denoted by (1,1). For the agent who has been unemployed for at least 2 periods, his employment status is denoted by (2,0). Note, when one is unemployed for more than 2 consecutive periods, he is not eligible for UI benefit anymore for the current unemployment spell.

In the economy, we have the following parameters to be assigned: σ, the coefficient of relative risk aversion; δ, the parameter in the disutility function of effort; y, output of the risk-free project; b, the UI benefit in terms of consumption good; λ, the UITF balance as a percentage of total taxable wages; θ, the eligibility probability; β, the discount factor; and the remaining three parameters of effort effectiveness: r_0, the one for agents who are employed last period; r_1, the effort effectiveness parameter of agents who have been unemployed for only 1 period; r_2, the effectiveness of effort for agent who has been unemployed for at least 2 periods. The UI benefit is set so that the replacement rate is 0.5. We set β to 0.99 for quarterly data. For λ, we use the data of UITF as a percent of Taxable wage for United States, i.e., $\lambda = 0.70125\%$. [5] y is normalized to 1. [6] For the coefficient of relative risk aversion, we set $\sigma = 1$ and $\delta = 2$. Given β, σ, and δ, the remaining four parameters are set to capture features of the labor force data. That is, we set these data so that the steady state outcomes match with the average unemployment rate, the fraction of unemployed agents collecting UI benefit, and the distribution of unemployment by duration of unemployment, in the CPS data.

In 2005-2006, about 35.4% of the unemployed population received UI benefits. For the same period, the average unemployment rate is 4.8%. Among the unemployed population, 66.55% of the unemployed had been unemployed for 13 weeks or less, 14.8% of the unemployed had been unemployed for 14-26 weeks, and 18.6% had more than 26 weeks of unemployment. To replicate the data, we set $r_0 = 9.61$, $r_1 = 1.81$, $r_2 = 0.64$, and $\theta = 0.429$. For the employed agents, effort is most effective in keeping the current job. Effort becomes less efficient in finding

[5]Interested readers can find the data of UITF as a percentage of the total taxable wages via the following link: http://www.workforcesecurity.doleta.gov/unemploy/content/data.asp.

[6]When y is normalized to 1, τ and b become the UI tax rate and the replacement rate. I do not intend to match the aggregate income to the U.S. data in this paper. Therefore, the level of y will not make a difference in terms of the characterizations of the solution.

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new jobs when periods of unemployment increases ($r_2 < r_1$).

3.4.2 Calibrated Solutions

The equilibrium UI tax rate and net interest rate are 0.089% and 0.7889%, respectively. I first look at the effort level of agents with different employment status, then their consumption and saving behavior. Note that the aggregate behavior of this economy is dominated by that of employed workers, since they amount to 95% of the whole population.

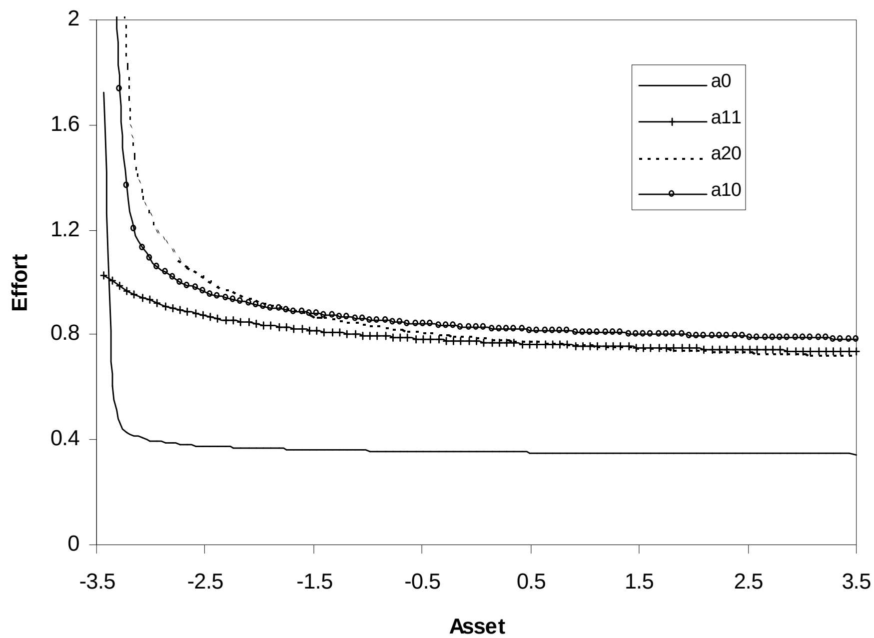

Figure 3.1 Effort functions

Fig. 3.1 plots the effort functions of all agents. Here, a_{jl} denotes the effort level of an agent who has been unemployed for j periods at the beginning of the current period and whose eligibility status is l. Each agent can use two means to smooth his consumption: either exert a high effort to gain a higher probability to be employed, or borrow (dissave) to finance consumption. The cost of the former is the disutility of effort, while the cost of the latter is an intertemporal substitution effect. Increasing today's consumption causes a reduction in tomorrow's consumption. As agents own more assets, being employed seems less attractive.

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Therefore, agents tend to rely less on expanding effort to smooth their consumption. This is illustrated by that the effort level is a decreasing function of agents' asset holdings. The comparison among the effort level of all agents shows that those employed exert the lowest effort. That is simply because their effort are most effective in locating a job. Among workers who are unemployed, $a_{10} > a_{11}$, that is, those who have been unemployed for 1 period and not receiving UI benefits make higher effort than those who receive UI benefits. This is clearly due to the income effect induced by UI benefits. Being unemployed does not seem so bad for agents who can claim UI benefits, and hence, they exert lower effort. We also notice that $a_{20} > a_{10}$ and $a_{20} > a_{11}$ for agents who bear large amount of debts. However, as agents bear less debts or hold more assets, it become such that agents in status (1,0) exert the highest effort. This is because these type of agents have the expectation that if they stay unemployed for one more period, their effort will become less effective in locating a job. Consequently, they would like to make higher effort now to be employed more likely.

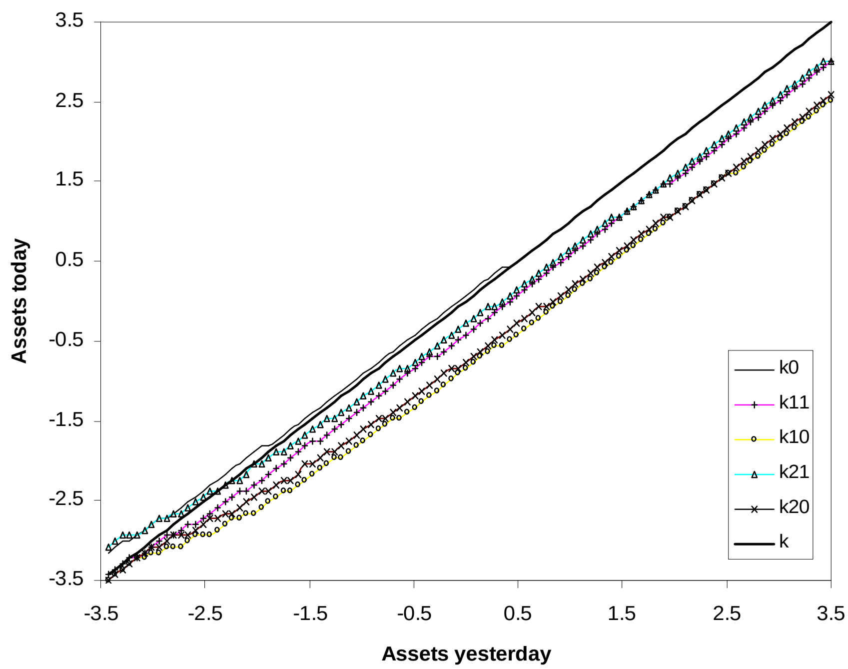

Figure 3.2 Law of motion for assets

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Fig. 3.2 shows the end-of-period asset of agents given their beginning-of-period asset holdings. Note that the time sequence of this decision happens at the end of the current period. Hence, jl here is different from the notation jl used for the effort function, which defines status at the beginning of the current period. For instance, k_{11} is the end-of-period asset of an agent who is employed last period but just becomes unemployed and eligible for UI benefits this period. In general, saving is an increasing function of agents' assets. And among all agents, employed agents borrow the lowest amount of debt or save the most. It is also noticed that k_0 lies above the 45 degree line up to 0.42, and then two lines overlap. It suggests that employed workers always accumulate more assets or borrow less than what they hold at the beginning of the current period. As agents become sufficiently rich, i.e., their asset holdings are at least of 0.42, they start to hold constant assets over time.

Next, we notice that $k_{21} > k_{11} > k_{20} > k_{10}$. This has three implications.

First, for the unemployed agents who are eligible for receiving UI benefits, those who have been unemployed for the second period hold less debts (save more) than those who have just become unemployed. It is the similar case for the unemployed workers who do not receive UI benefits. The reason is that effort is less effective when they are unemployed longer, i.e., $r_2 < r_1$. Agents who have been unemployed for 2 periods expect that they are more likely to remain unemployed in the future. Therefore, these agents would like to save more or borrow less for future consumption if unemployed.

Secondly, agents who receive UI benefits save more than those not receiving benefits. Agents who remain unemployed in their 2nd period have the same expected future utility disregarding their eligibility status. Therefore, agent who receive benefits are able to save more because of income effect. The intuition is a bit different for those who just become unemployed. An additional saving today will matter more for those who are not eligible for UI benefits. Thus those workers tend to save more. However, workers also tend to save more when they receive benefits due to the income effect. It turns out that the income effect dominates the other one. As a result, agents who just become unemployed and yet receive benefits save more than those not receiving benefits.

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Lastly, the difference in the saving amount between workers who are in their 1st period of unemployment spell and 2nd period is diminishing as agents possess more assets. Obviously, when agents have more assets, the marginal benefits brought by the saving today become smaller. As a result, agents' saving is increasing in a smaller way.

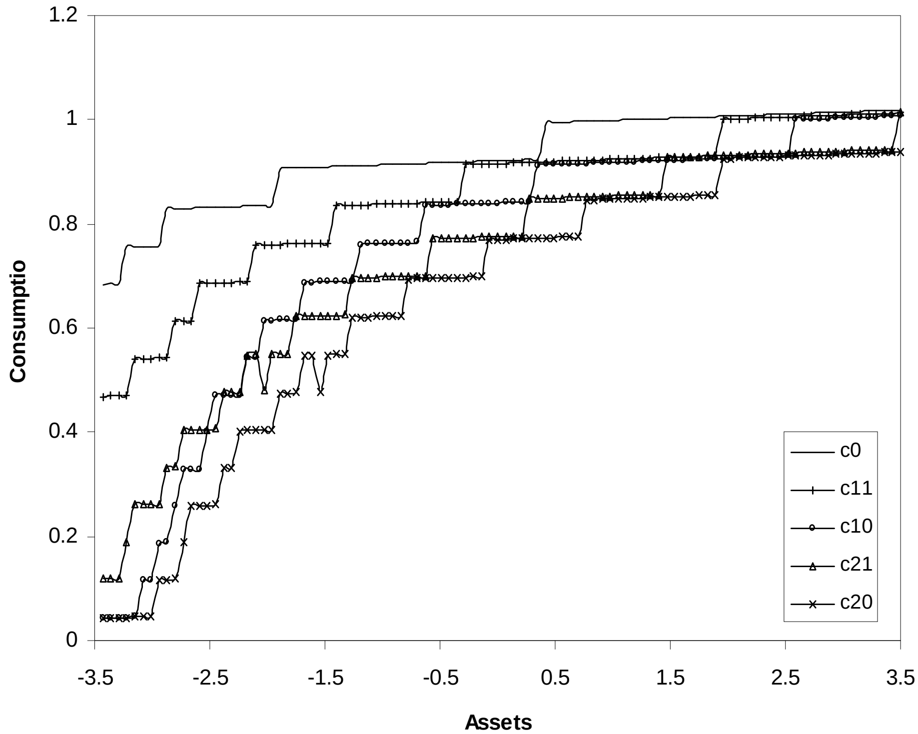

Figure 3.3 Consumption functions

Fig. 3.3 shows the consumption paths of all agents across employment status and asset holdings. Agents consume more good as they possess more assets. Those who are employed consume the most because they receive the highest income. When a worker first become unemployed, his consumption will be reduced by 93% if he possesses the largest debt, and only 1% if he is very rich conditional upon not receiving UI benefits. If he receives UI benefits, the drop in the consumption is getting smaller, around 32% even when he owes the largest debt. If he stays unemployed again for the next period, however, the consumption-smoothing effect brought by the UI is getting smaller. The reduction in the consumption level rises back to

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around 90%. This refers to the short-term consumption smoothing effect of UI program shown in Gruber (1997). Also note that the difference in the consumption level between the agents receiving UI benefits for the 2nd period and those becoming unemployed and yet not receiving UI benefits is different across asset holdings. The former consume more than the latter when they are sufficiently poor, while less when they become less poorer. That is because the reduction between two types of agents' savings dominates the increase in their labor income, UI benefits.

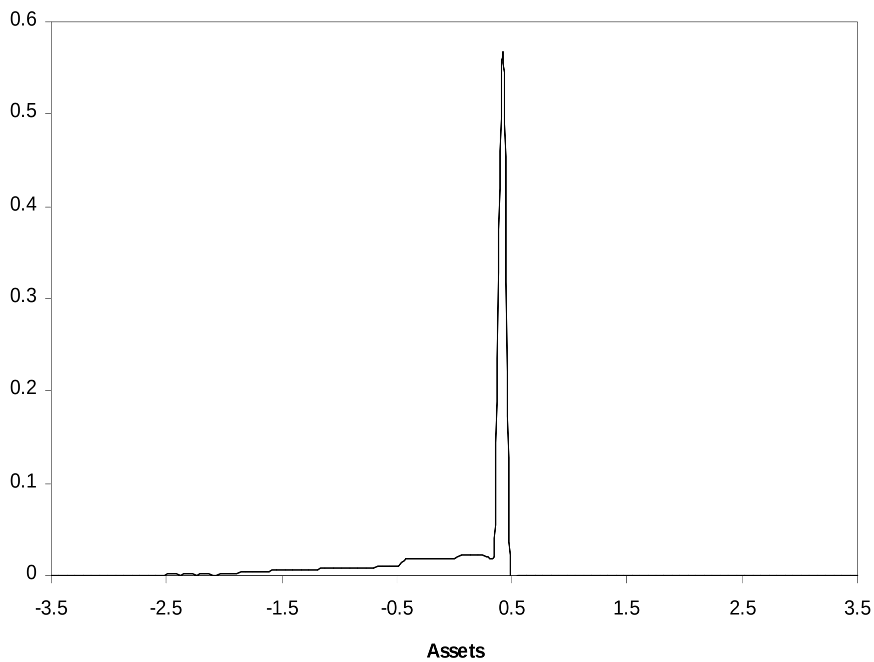

Figure 3.4 Distribution of wealth for employed agents

Fig. 3.4 displays the distribution of employed agents over their asset level. For most of the employed agents, their assets are concentrated at 0.42. This corresponds to the level before which the saving decision is above the 45 degree line, illustrated by Fig. 3.1. That means an employed agent with low asset holding or high debts always save or borrow less. Hence his asset is moving upward till the point at which the saving is constant over time. Since only 5% of the agents are unemployed in the economy, I do not display their distribution. The highest

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level of asset an unemployed agent hold is zero, which means that all the unemployed workers bear debts. Among them, it is the agent who receives UI benefits upon unemployment owes the lowest amount of debt, and the one not receiving UI benefits for his initial unemployment owes the largest debt.

3.5 Policy Experiments

In this section, I have computed the stationary equilibrium solutions for two policy experiments. The first is to set the UITF balance to 0. The second economy is to remove the UI program and set the UITF balance to 0, which implies that $b = 0$ and $\tau = 0$. I then compare the solutions among these three economies.

3.5.1 Baseline Model vs. UITF$= 0$

I first study the effects of a positive UITF balance when a UI program is at presence. The first 2 rows of the Table 1 lists the equilibrium tax rate, interest rate and the welfare of the whole economy. A positive UITF balance increases the UI tax rate. In addition, since it crowds out the private savings in the credit market, the capital interest rate also rises. It turns out that the current UITF balance improves the welfare by 0.65%.

A positive UITF balance also has impacts on individual's behavior, although the magnitude of the effects is rather small, ranging from a low 0 to a high 0.05%. The numerical results suggest that both employed and unemployed agents increase their job searching effort level, and there is an increase for both types of agents in their saving decisions. This further implies that there is a fall in agents' consumption today. The positive UITF balance also affects the distribution of agents across the asset holdings. Majority of the employed workers now possess higher assets, or they become richer, which explains why a positive UITF balance improves welfare.

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3.5.2 Baseline model vs. No UI

Since the change induced by the positive UITF balance is quite small, I thus skip the comparison between the economy without the UITF and the economy without a UI program. In this section, I study how the economy changes when a UI system is not available.

3.5.2.1 Costs of UI

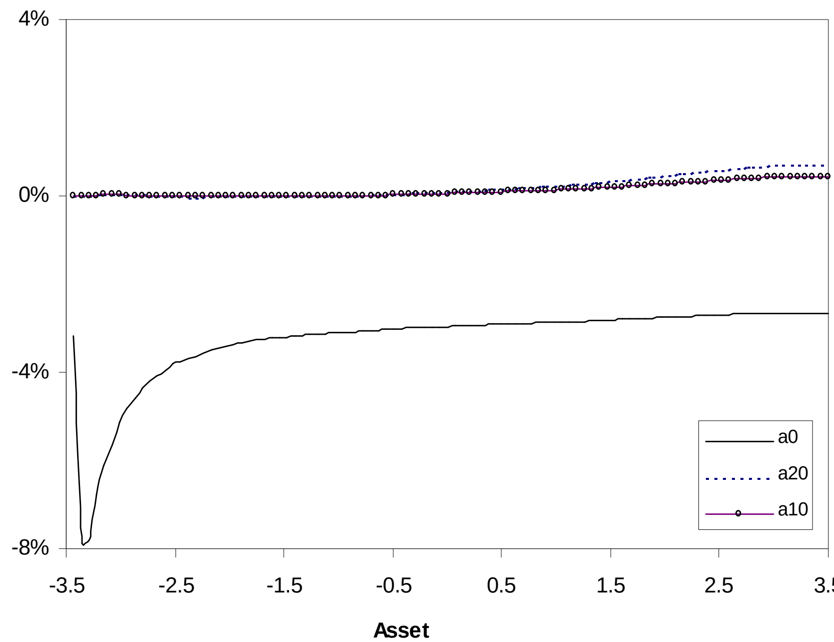

Figure 3.5 Change in the effort level of employed agents, unemployed agents (1,0) and (2,0)

Fig. 3.5 and Fig. 3.6 present the percentage changes in the effort level of unemployed agents and employed agents respectively, when the current UI is removed. In general, agent exert lower search effort when the public UI system is available, supporting the well-know statement that UI distort workers' job searching behavior. The fall in the effort level for agents (1,1) is the largest, ranging from 41.76% to 6.94%. Employed workers experience a relatively smaller drop in their effort, from 8% to 3%.[7] UI has almost no effect on unemployed

[7] Notice that there is a kink for the percentage change in effort. It starts to drop first and then increase.

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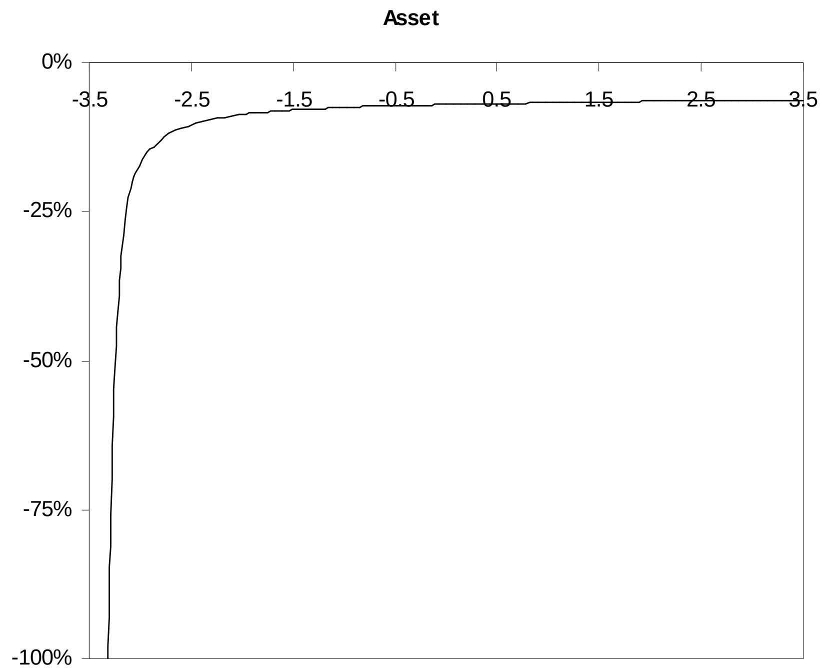

Figure 3.6 Change in the effort level of agents (1,1)

workers who do not receive UI benefits and bear high debts. However, quite unexpectedly, unemployed workers with very high asset holdings expand a higher effort in the baseline model by 0.79%, comparing to what they would do without a public UI. However, this effect does not appear at the aggregate level, since very rare unemployed agents possess such high level of assets.

Fig. 3.7 and Fig. 3.8 illustrate the percentage changes in the saving decisions for all agents. Employed agents experience a small drop (increase) in their asset (loan) holdings, probably between 0% and 8%. Public savings crowd out employed agents' private savings. This is because UI system affects an employed worker in two ways. First, an employed worker's labor income is reduced because he has to pay for UI tax. Consequently, he would like to lower his savings. Second, agents use their own savings as an insurance against the risk of income loss due to unemployment. However, when a public UI becomes available, the income loss is

This is purely due to the calculation of the the percentage change. The effort level at the starting point is quite high, about 1.72, relative to 0.6, the effort level as assets start to rise. Hence, for the same level of variation, the percentage change is getting quite small.

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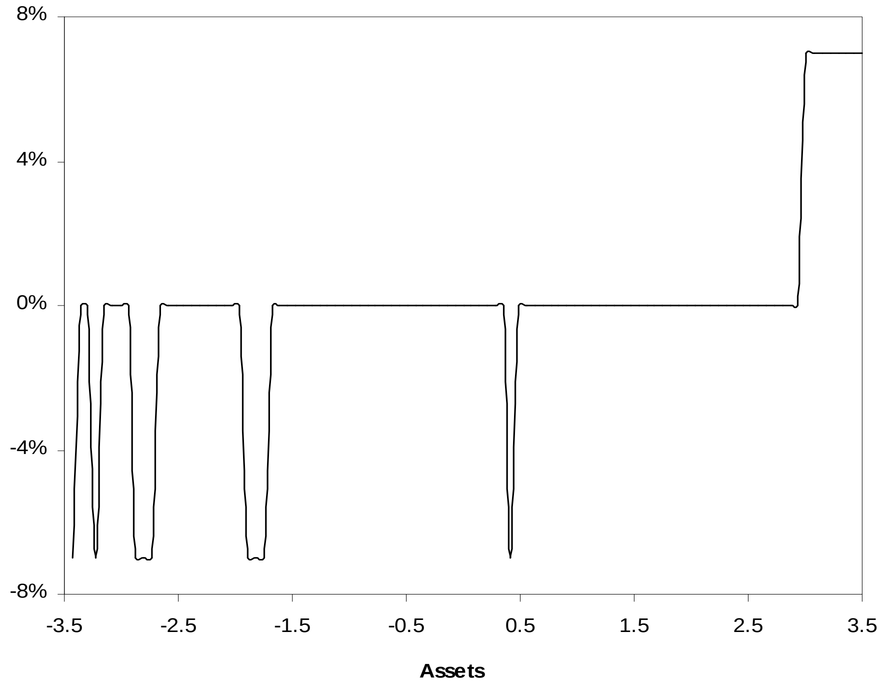

Figure 3.7 Changes in the savings of employed agents

now reduced and as a result, an agent will rely less on his own self-insurance scheme. These two effects combine to lower an agent's asset holdings for the future. It appears that the UI actually increases the savings of employed agents when they are sufficiently rich.

In contrast to the drop in the savings of employed workers, unemployed workers increases (decreases) their savings (debts) when they are eligible for UI benefits. Agents who are in the 2nd period of their unemployment spell experience a relatively large increase in their saving level, ranging from 28% to 49%. This is mainly due to the income effect. Agents are now being able to accumulate more for future consumptions. For those agents who have just become unemployed, their debts level are not affected much, when they hold large debts. This is because the UI benefits do not help them much in relieving the heavy debts burden. Notice that this also corresponds to the fact that, there isn't much different for the saving behavior of agents (1,1) and (1,0) in the baseline model, illustrated by Fig. 3.4, when they hold large amount of debts. However, as agents possess more assets, they experience an increase in their savings, similar to agents who are in the 2nd period of their unemployment spells.

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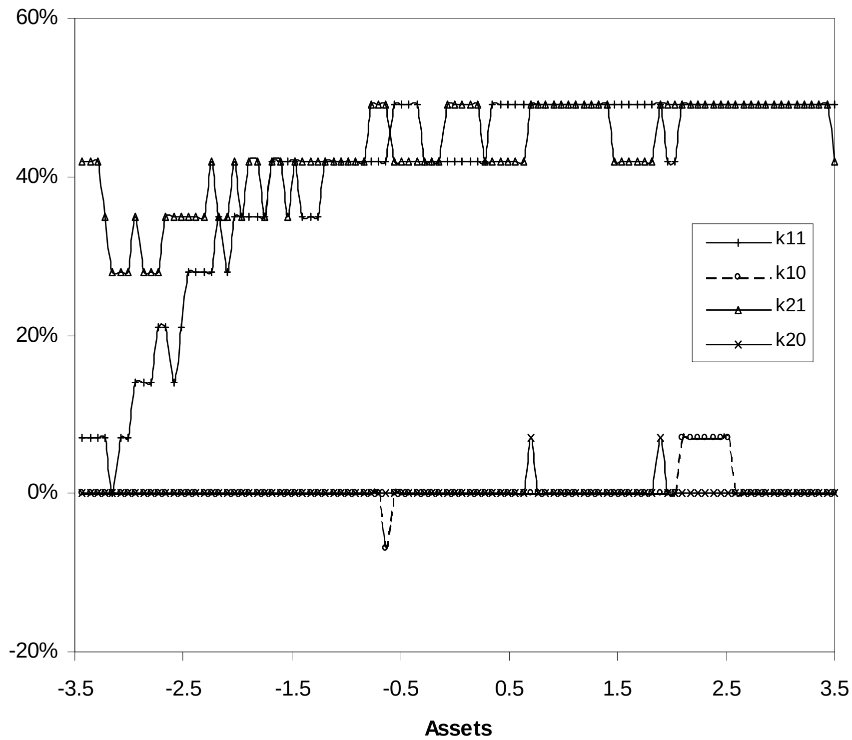

Figure 3.8 Change in the savings of unemployed agents

3.5.2.2 Benefits of UI

Fig. 3.9 displays the percentage changes in the consumption level for employed agents when a public UI is unavailable. Employed agents consume less with the introduction of the current UI program other than few exceptions (corresponds to the change in the saving behavior). The reason for this is that employed agents' total income have been largely lowered by the UI tax even though they enjoy a higher capital interest rate. Due to the income effect, employed agents consume less goods. Note that an employed agent who possesses very high assets experiences a relative larger decrease in their consumption level. This corresponds to the large increase in the savings, illustrated by Fig.3.7.

Fig. 3.10 lay out the percentage changes in the consumption for unemployed agents. Unlike employed workers, those who receive UI benefits experience a large rise in the consumption level. The rise can be ranged from 93%, when agents bear large amount of debts, to almost 0,

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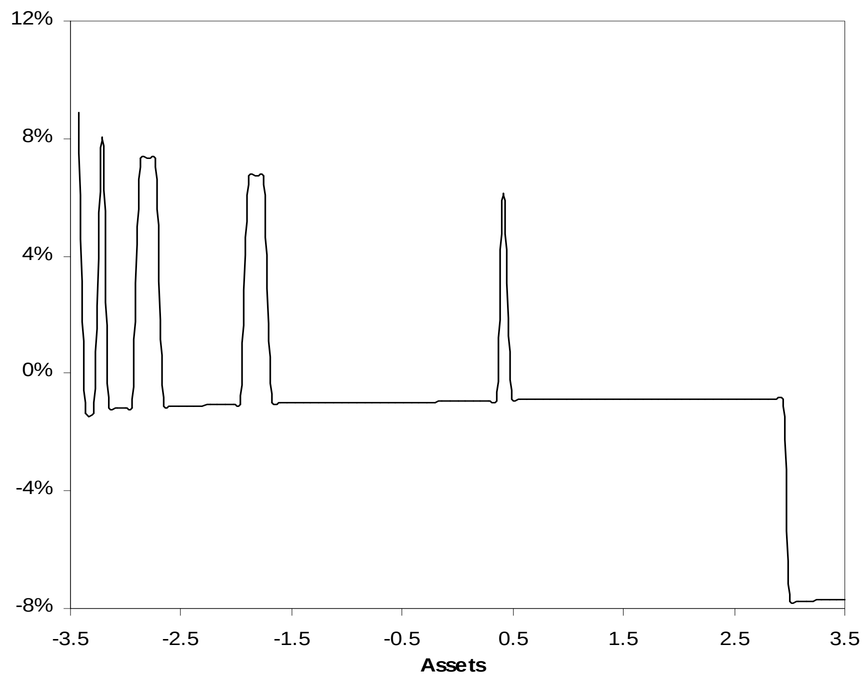

Figure 3.9 Change in the consumption of employed agents

for sufficiently rich agents. In contrast to the unemployed workers receiving UI benefits, those who do not receive benefits endure almost no change in their consumption when UI is taken away. The exception is for workers whose savings range from 2.1 to 2.52. Those workers save less and hence consume more when there is no public UI provision. Fig. 3.11 displays the distribution of employed agents across assets for three different economies. For the baseline model, more than half of the employed workers hold assets of 0.42, and the rest either bear debts or hold lower assets. This is consistent with the law of motion of assets. Employed agents bear less loans or hold more assets than their current asset holdings up till 0.42. The current system induces the highest unemployment rate, while the lowest unemployment rate, 4.2%, is achieved in the economy where the government UI is not available. Comparing the distribution of employed agents between two economies, we find that the employed agents are able to accumulate more assets in the environment when a public UI is unavailable. In other words, we expect to observe the highest assets holdings in the economy without UI system.

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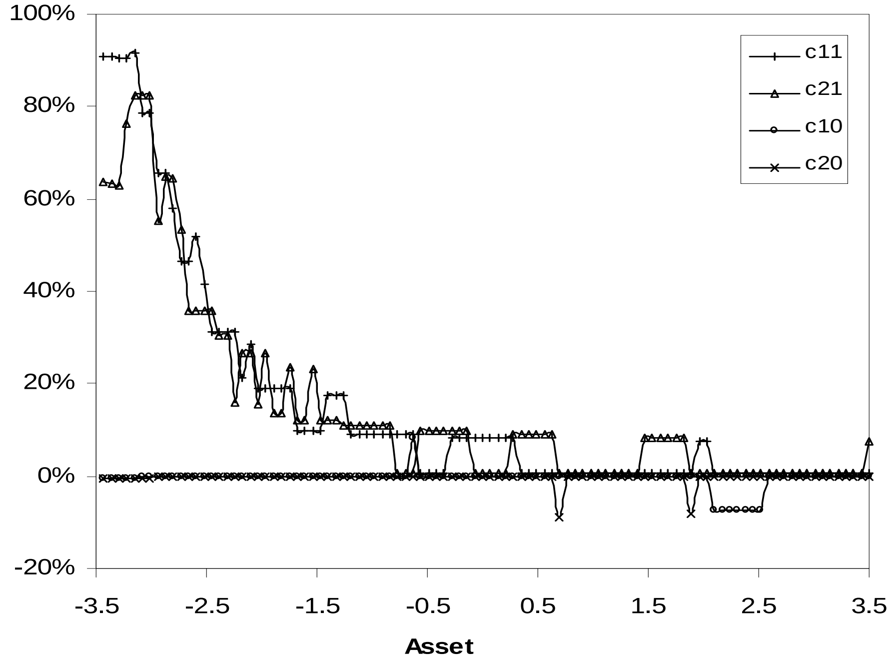

Figure 3.10 Change in the consumption of unemployed agents

3.5.2.3 Aggregate Performance

Table 3.1 Interest Rate and Tax Rate

	Interest Rate	Tax Rate(%)	Welfare
Baseline	1.007889	0.89%	-20.9796
NoUITF	1.007881	0.88%	-21.0148
NoUI	1.007812		-20.7589

The last row of Table 3.1 lists the interest rate, tax rate and the welfare for the economy when a UI program is unavailable. The current model generates the highest interest rate and the UI tax rate. UITF has a relatively small effect on the interest rate and tax rate. The economy without a UI program generates the highest welfare. The cost of the UI system dominates the consumption-smoothing benefits. Table 3.2 presents the total measure of unemployed workers in three different economies. The economy without a government UI provision has the lowest number of unemployed workers, 0.04246. This is because a large part of the population

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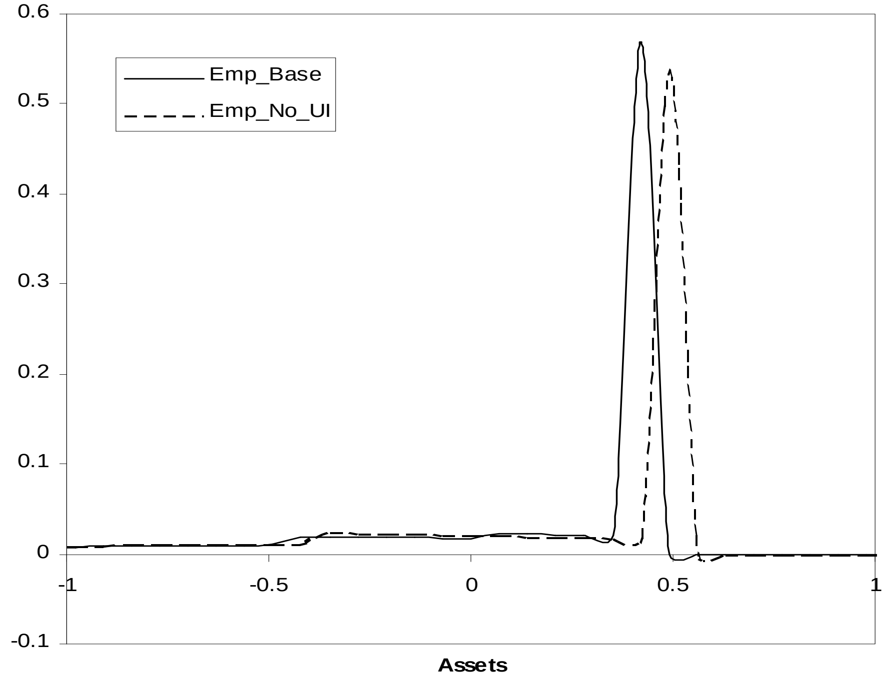

Figure 3.11 Distributions of wealth for employed agents

Table 3.2 Components of Unemployed

	(1,1)	**(1,0)**	**(2,0)**
Baseline	0.01362	0.01818	0.01619
NoUITF	0.01361	0.01812	0.01608
NoUI		0.02890	0.01356

is composed of employed workers, and employed workers exert the highest effort when UI is absent. In general, 2/3 of the unemployed workers are only unemployed for only 1 period, and 1/3 have been unemployed for at least 2 periods. Among three model economies, the economy without the UI system has the lowest number of agents who have been unemployed for at least 2 periods, while the baseline economy has the highest. That is largely because agents (1,1) exert lower effort when UI is available.

Table 3.3 displays the total savings for agents with different employment status across three economy settings. Employed workers save the most in the economy with no UI and the least for the baseline model. Unemployed agents borrow more when UITF balance is zero than the

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Table 3.3 Aggregate Assets

	(0,1)	(1,1)	(1,0)	(2,1)	(2,0)	(3,0)
Baseline	0.03793	-0.00439	-0.01431	-0.00186	-0.00550	-0.01855
NoUITF	0.04705	-0.00521	-0.01536	-0.00204	-0.00569	-0.01875
NoUI	0.04790	0.00	-0.02220	0.00	-0.00860	-0.01710

amount they would borrow for the baseline model. It is also true when we compare the loans made by unemployed workers between the model with no UI and that with no UITF. In other words, a public UI system crowds out private savings and a positive UITF balance exaggerate the crowding out effect.

Table 3.4 Aggregate Consumption

	Welfare Gain	**(0,1)**	**(1,1)**	**(1,0)**	**(2,1)**	**(2,0)**	**(3,0)**
Baseline		0.9145	0.0123	0.0158	0.0025	0.0025	0.0045
NoUITF		0.9144	0.0123	0.0157	0.0025	0.0025	0.0044
NoUI		0.9253		0.0251		0.0040	0.0033
NoUI v B	0.65%	1.18%		-10.65%		-19.84%	-26.39%
NoUITF v B	-0.65%	-0.01%	-0.24%	-0.28%	-0.40%	-0.79%	-1.77%
NoUI v NoUITF	1.79%	1.19%		-10.36%		-19.35%	-25.06%

The first column of Table 3.4 compares the welfare change across three economies. Apparently, agents in the economy with no UI system consume more than what they would in the model without a UITF by 1.79%, according to the standard compensated variation measure.[8] This welfare gain is slightly higher than 1.63% in Eric Young (2004) and larger than -0.09% in Wang and Williamson (2002). Eric Young (2004) allows agents to borrow subject to an endogenous liquidity constraint. However, the UI program has an additional effect such that the aggregate output decreases due to a lower level of aggregate savings, or lower capital inputs

[8] In detail, this percentage of the increase in consumption is calculated using the following equation

$$W_1 = W_0 + \frac{1}{1-\beta}\log(1+\phi)$$

where W_0 is the expected utility in the baseline model and W_1 is the expected utility when UI is removed. This equation is provided by Aiyagari and McGrattan (1998).

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in the production. The welfare gain in this model maybe higher if we also incorporate the changes in capital input. Wang and Williamson (2002) allows agents to save, but only in the form of a non-interest-bearing asset and finds that the existing UI improves welfare. Not allowing agents to borrow may contribute to the welfare gain induced by the existing UI system in Wang and Williamson (2002). The next 6 columns of the Table 4 decompose the total measure and the percentage change in the consumption across different agent groups. For unemployed agents, those in the baseline model consume the most for the baseline model. We first look at what happens when there is no public UI system. Employed workers enjoy a gain of 1.18% in their consumption level, while unemployed workers experience a relatively large drop in their consumption, ranging from 10% to 26%. The similar patterns occurs when we compare the same terms between the economy with no UITF and no UI. However, we notice that both employed agents and unemployed agents experience a drop in their consumption level when UITF balance is removed from the economy while a public UI system is still present. This is consistent with the results that a positive UITF balance reduces the welfare, and the lack of a public UI improves the welfare (see Table 3.1).

3.6 Conclusion

In this paper, I investigate the costs and benefits of the current UI program in an environment where agents are able to borrow and lend and the government holds a positive UITF balance. The ability to borrow and lend provides agents with a better self-insurance scheme against the risk of income loss. In most of the previous studies, agents either are not allowed to save, or they can only save in the form of storage. However, even though agents can borrow to finance their consumption, the intertemporal substitution effect still exists. A higher debt today increases today's consumption in the cost of a lower consumption tomorrow. Therefore, the motive for the precautionary savings exists as well. A positive UITF balance affects the economy in the following ways. First, it can be viewed as forced public savings. This would have a crowding-out effect for the private savings, which further lowers the interest rate. Hence, the government is able to affect agents' behavior in an indirect way by adjusting the balance

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level. Second, the current UITF balance increases the tax burden levied on employed agents. It turns out that a public UI program and a positive UITF balance work together to reduce the income loss if an agent becomes unemployed.

I use the numerical results to analyze the effects of the UI system and the UITF balance. I find that the current positive UITF balance actually increases welfare in the presence of a UI. This is mainly because employed agents are able to accumulate more assets, and hence, the consumption level enjoyed by majority of the agents increases. The second finding is that, when a public UI is unavailable, employed workers always save until they are sufficiently rich, and unemployed workers always borrow to finance their consumption. The introduction of a UI program causes unemployed agents to borrow less and hence the total savings in the economy is now reduced by 1.78%. A positive UITF balance further exaggerates the crowding-out effect, reducing the total savings by 6.87%. Last, the economy with a pure self-insurance scheme generates the largest welfare, since there are more employed workers and they enjoy the highest consumption among all agents.

It is probably useful to mention that this analysis may have little values in policy implication. First, many empirical evidences have shown the existence of a liquidity constraint. In fact, many people are not allowed to borrow at all. In order to make use of the results, we need to explore more about how the data suggests for the self-insurance scheme. Second, the UITF balance does not have any real value here. It cannot be used to increase agents' consumption. In practice, it serves as a back up reserve for the increases in UI claims when recession happens. Since there is no aggregate shock in this paper, the welfare benefit of a positive UITF disappears. In order to better judge the effects of the UITF, a business cycle setting is probably more appropriate. However, this is beyond the extent of this paper and I will leave it to future analysis.

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CHAPTER 4. PUBLIC AND PRIVATE EXPENDITURE ON HEALTH IN A GROWTH MODEL

4.1 Abstract

This paper introduces endogenous longevity in an otherwise standard overlapping generations model with capital. In the model, a young agent may increase the length of her old age by incurring investments in health funded from her wage income. Such private health investments are assumed to be more "productive" if accompanied by complementary tax-financed public health programs. The presence of such a complementary public input in private longevity is shown to expose the economy to aggregate endogenous fluctuations and even chaos, and such volatility is impossible in its absence.

Keywords: longevity, public health, chaos

JEL Classification: E10, J10, O10,

4.2 Introduction

According to the World Health Organization, the life expectancy at birth in 2003 for Swaziland was 35 years; the same for Japan was 82 years.[1] In 2002, the life expectancy at age sixty in Sierra Leone was 8 years; that in Japan was nearly 20 years.[2] These stupendous gaps in longevity between countries present a challenge to epidemiologists and economists.

[1]The data are taken from http://www.who.int/entity/whr/2005/annex/annex1.xls. The gap between the highest ten life expectancy countries and the lowest ten is about 40 years. Furthermore, as discussed in Murray and Lopez (1994), on average a new born child in the "Established Market Economies" has only a 1% chance of dying before reaching age 15 which is much lower than the 25% risk in Sub-Saharan Africa.

[2]In 2003, only 4% of the population in Swaziland were of age 60 or higher compared to 25% in Japan. Murray and Lopez (1994) point out the probability of dying for a person in the 15-60 age bracket was upwards of 30% in many of the less devloped parts of the world (such as Sub-Saharan Africa) and less than 13% in the developed countries. A large part of the difference can be attributed to differences in incidence of communicable diseases, an area in which public health programs have been particularly important and effective.

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In 2000, the World Health Organization (WHO, 2000) released a major report in which it argued that "the differing degrees of efficiency with which health systems organize and finance themselves, and react to the needs of their populations explain much of the widening gap in death rates [..] between countries..." In effect, the report argued that longevity differences between countries had a lot to do with the economics of health systems and much less about medicine (see Navarro, 2000).[3]

Health systems have two components, a public and a (often ignored) private one. As classified in the World Development Report (1993), public health programs "work in three ways: they deliver specific *health services to populations* (for example, immunizations), they promote healthy *behavior*, and they promote healthy *environments*".[4] That same report also eloquently argues that "what people do with their lives [...] affects their health far more than anything governments can do".[5][6] This paper presents a simple framework to study this dynamic complementarity between public health programs and private efforts to improve longevity.

To that end, the paper presents an otherwise standard overlapping generations model with production (a la Diamond, 1965) modified to include longevity concerns. In the model economy, young agents work for a competitive wage and each survive to the start of old age (the second period). However, any agent is alive only for a fraction of the second period of her life. A young agent may increase the length of her old age by incurring investments in her own health funded from her wage income. All agents care only about their old age consumption while alive. Hence the allocation problem for young agents is one of deciding how much to

[3]The same WHO report presents the message as follows: "If Sweden enjoys better health than Uganda — life expectancy is almost exactly twice as long — it is in large part because it spends exactly 35 times as much per capita in its health systems."

[4]The World Development Report (1993) documents that nearly 60% of world spending on health (roughly 8% of the world's income) comes from governments. In developing countries, government spending from general tax revenues amounts to roughly half of the $2 - 7\%$ of GNP allocated to public health.

[5]Private efforts to improve health and longevity may include annual diagnostic health screening, opportunity cost of regular exercise, taking vitamins, nutrients, and other supplements, eating organically grown food, health benefits from quitting unhealthy habits such as smoking, etc. In developing countries, these may also include some out-of-pocket expenses for essential medication or clinical services provided by an (often) unregulated private health sector. As Fabricant, Kamara, and Mills (1999) document, such expenses range from $2\% - 5\%$ of household income among the upper income groups in India, Vietnam, Bangladesh and Nepal to as high as $10 - 25\%$ for the poorer quintiles in Azerbaijan and other countries.

[6]Similar sentiments ("individual vs. social responsbility for health") are echoed in the United States starting with the 1979 Surgeon General's Report on Health Promotion and Disease Prevention.

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invest in one's longevity and how much to save for the future. Our assumptions imply that longevity and youthful savings are both normal goods. We assume that tax-financed public health programs exist whose main purpose is to complement the private health investments.[7] Specifically, we assume that the marginal impact on longevity of a marginal increase in private health investment rises with increased public expenditure on health.

Our main results are as follows. Under some mild assumptions on the utility and the longevity function, we can prove the existence of a unique non-trivial steady state capital-labor ratio. We go on to study the dynamics of the law of motion for the capital-labor ratio. Under the assumption that public spending on health is a convex function of the government's tax collections, we can show that the time-map of the capital-labor ratio is unimodal and non-monotonic. We are able to demonstrate the presence of endogenous volatility in a small neighborhood of the steady state capital-labor ratio. We show that a necessary condition for such fluctuations is that the private and public inputs into longevity be complementary. By means of a reasonable numerical example, we are also able to verify that our time map satisfies a set of sufficient conditions for topological chaos outlined in Mitra (2001). The upshot is that tax-financed public expenditures aimed at enhancing individual agent longevity may introduce endogenous volatility in an economy where such fluctuations are *impossible* in their absence.

Our paper is in line with a new yet burgeoning literature that incorporates mortality concerns in growth models. The seminal papers in this literature are Blackburn and Cipriani (2002) and Chakraborty (2004). Blackburn and Cipriani (2002) endogenize old-age mortality concerns in a growth model. They assume that in a three-period overlapping generations model, the probability with which an agent survives to the third period depends on the stock

[7] We do not wish to interpret the public component as a competitor to the private input in health markets; viewed in this way, the two inputs would have to seen as being substitutes in health service provision. Instead, we are stressing the complementarity in investments, i.e., we view public health investments as being complementary to private health investments.

We are underscoring the government's role in financing and implementing programs that provide direct or indirect health benefits/externalities to private citizens; direct efforts such as anti-pollution programs that improve air quality and raise longevity for all, or indirect efforts such as anti-smoking programs that increase awareness of the health costs of first and second-hand smoking, thereby inducing some individuals to quit the habit.

An example of our sense of the complementarity between the private and the public inputs in longevity is the following. Private efforts to raise one's health stock via regular exercise etc. may generate huge improvements in longevity if concomitantly the government finances public efforts to reduce atmospheric pollution.

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of human capital or more generally, on the level of development of the economy itself. They use their framework to study issues relating to the demographic transition, as well as the effect of longevity on economic activity. Their model generates multiple development regimes; in particular, there exists a threshold initial human capital stock below which the economy ends up poor. Chakraborty (2004) is an exploration of a new connection between pervasive ill-health and economic growth. He introduces endogenous mortality in an otherwise standard overlapping generations model with production; in particular, the probability with which a young agent survives on to old age depends on public health expenditures which are in turn funded by income taxes on labor income. Like Blackburn and Cipriani (2002), Chakraborty shows that if the starting capital stock is sufficiently low, the country is more likely to be permanently stuck in a poverty trap. Finlay (2005) and Haaparanta and Puhakka (2004) generalize the environment and broaden the scope of the results in Chakraborty (2004).[8] Aisa and Pueyo (2004) produce a continuous time overlapping generations model where tax-financed public revenues can provide a production externality or improve social health (average longevity). In their setup, a longer life expectancy results in an increase in savings (as in Chakraborty, 2004); it also increases the available workforce and may increase growth and even average longevity.

The novelty of our work relative to these papers lies in our exclusive focus on the interaction between the public and private component of health systems and its consequences for aggregate volatility. Indeed, our result on the existence of complex dynamics (chaotic equilibria) is of some independent interest. There is a vast literature studying the possibility of complex dynamics in general equilibrium growth models, especially in overlapping generations models.[9] As is well-known in this literature, complex dynamics can emerge under assumptions of limited market participation, imperfect competition, multiple sectors etc. Additionally, as discussed in Azariadis (1993), a sufficiently strong income effect which, in turn, causes savings to decline with the interest rate and produces "backward-bending" savings functions can also produce complex dynamics in overlapping generations models. In our model, chaotic dynamics emerge

[8] Our analysis and its focus differs from that in Chakraborty (2004) in the way that we allow longevity to be chosen optimally by private agents themselves.

[9] See Lorenz (1993) and Mitra, Majumdar, and Nishimura (2000) for surveys of this literature.

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in a relatively standard economy; indeed, given our assumptions on preferences, optimal saving in capital is independent of the return on capital, and without the public input in private longevity, our model economy would not produce endogenous fluctuations of any kind. Thus, ours is not *yet* another paper demonstrating the presence of complex dynamics in neoclassical growth models. Our novelty lies in its emphasis of the role of endogenous longevity and the interaction between the two components of health systems in generating these complex dynamics.

The rest of the paper is organized as follows. In Section 4.3, we present the details of the specification of the model economy and derive the solution to the agents' problem. Section 4.4 presents our main results concerning the general equilibrium law of motion for the capital-labor ratio while Section 4.5 concludes. Proofs of major results are in the appendix.

4.3 The Model

4.3.1 Environment

We consider an economy consisting of an infinite sequence of (potentially) two-period lived overlapping generations, and an infinitely-lived government. Let $t = 1, 2, ...$ index time. At each date $t > 1$, a new generation comprised of N identical members appears. Each agent is endowed with one unit of labor when young and is retired when old. In addition, the initial old agents are endowed with $k_1 > 0$ units of capital.

There is a single final good produced using a standard neoclassical production function $F(K_t, L_t)$ where K_t denotes the capital input and L_t denotes the labor input at t. The final good can either be consumed in the period it is produced, or it can be stored to yield capital at the very beginning of the following period. Capital is assumed to depreciate 100% between periods. Let $k_t = \frac{K_t}{L_t}$ denote the capital-labor ratio (capital per young agent). Then, output per young agent at time t may be expressed as $f(k_t)$ where $f(k_t) = F(\frac{K_t}{L_t}, 1)$ is the intensive production function. We assume that $f(0) = 0$, $f' > 0 > f''$, and that the usual Inada conditions hold.

Let c_{t+1}^t denote the consumption of the final good by a representative old agent born at

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t. All such agents have preferences representable by the utility function $U(c_{t+1}^t) = E_t u(c_{t+1}^t)$ where $u : \Re_+ \rightarrow \Re_+$ is twice-continuously differentiable, strictly increasing, and strictly concave in its arguments.[10] We will specialize to the commonly used form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $0 < \sigma < 1$.

All young agents survive to old age (the second period). However, any agent is alive only for a fraction $\theta \in (0, 1]$ of the second period of her life. In this sense, $1 + \theta$ captures the notion of longevity in our model.[11] Agents may influence their longevity by undertaking private investments (x) in their own health when young.[12] These investments have to be privately funded from wage income (see below). Following Dranove (1998), one could interpret these expenses as "preventive" medicine to prevent "premature death".

We assume that θ is strictly increasing and strictly concave in x. For much of what we present below, we assume a simple constant-elasticity functional form for θ:

$$\theta(x; \eta) = b\eta x^{b\eta};\ b, \eta > 0,\ \theta \in (0, 1] \tag{4.1}$$

where b is a parameter and η is the endogenous public input in private longevity (discussed below). We require $\theta \in (0, 1]$ in all of what follows. Strict concavity of the function $\theta(x)$ implies the restriction $b\eta \in (0, 1)$.[13] Following common practice in the literature, one can also interpret $\theta(x)$ as a "longevity production function". The elasticity of longevity with respect to private investment in longevity is then $b\eta$.

The government imposes a distortionary tax t on agents' incomes. If w_t is the wage rate, then the government raises revenue of amount $p_t \equiv tw_t$.We assume that η is a function of p_t, i.e., $\eta = \eta(p_t)$. We assume that $\eta(p_t) \geq 1$; if $\tau = 0$, then $p_t = 0$ and $\eta(0) = 1$; in this case, θ is described by $\theta(x) = bx^b$ which captures the *pure* effect of private investments in longevity. Henceforth, following Chakraborty (2004), we will label p_t as the public investment in longevity. Given that $\eta \geq 1$, the condition for strict concavity of $\theta(x)$ is now further reduced to $b \in (0, \frac{1}{\eta})$.

[10]See Hall and Jones (2004) for a discussion on why $u > 0$ has to hold and for anomalies with utility functions "multiplied with life expectancy".

[11]It is in this sense that ours is a model of longevity in old age as opposed to child or infant mortality. Our formulation has the added advantage in that it allows us to abstract away from issues relating to unintended bequests of the dead. These concerns are addressed in Aisa and Pueyo (2004).

[12]For a description of these investments, see Footnote 5.

[13]Values of b that are small enough help to ensure $\theta \in (0, 1]$ and $b\eta < 1$, thereby allowing us to avoid imposing conditions on x.

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The formulation in (4.1) nicely captures the essence of the complementary interaction between the public and the private components to health systems. If private efforts to improve health are not forthcoming, then a superior public health system does not help much in raising longevity, and vice versa. Notice from (4.1) that

$$\frac{x\theta'(x)}{\theta(x)} = b\eta = b\eta(p_t) \tag{4.2}$$

In other words, the elasticity of longevity with respect to private investment in longevity is itself influenced by public expenditures in the health sector. Additionally, the marginal impact of a marginal increase in private health investment on longevity is increased with more public expenditure on health. The better and larger the public health system, the larger is the impact of additional private investment on agents' longevity. As will also be clear below, the formulation in (4.1)-(4.2) provides a lot of analytical tractability.

This interactive formulation may be motivated roughly as follows. In a country with high environmental pollution or unsafe drinking water supplies, citizens are at great risk of catching debilitating illnesses such as severe allergies or asthma or even life-threatening diseases such as many communicable diseases, respiratory illnesses, and cancer. In such an environment, regular exercise and adherence to healthy diets may improve one's health but the overall harmful effects of pollution may still preclude a long and healthy life. Finally, as argued in Chakraborty (2004), "public health expenditure in new medical facilities, sanitation improvements, disease control and inoculation programs augments private health capital by reducing the economy-wide risk of contacting fatal diseases." Such complementarity is also stressed by Dow, Philipson, and Sala-i-Martin (1999), and in the context of young children, by Liu and Neilson (2005).

4.3.2 Trade and Markets

Young agents supply their labor endowment inelastically in competitive labor markets, earning a wage income of w_t at time t, where

$$w_t \equiv w(k_t) = f(k_t) - k_t f'(k_t). \tag{4.3}$$

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Capital is traded in competitive capital markets, and earns a gross real return of R_{t+1} between t and $t+1$, where

$$R_{t+1} = f'(k_{t+1}). \tag{4.4}$$

For $f(k) = Ak^{\alpha}$, it follows that $w(k) = (1-\alpha)Ak^{\alpha}$ and $R(k) = A\alpha k^{\alpha-1}$. For future reference, note that $w'(k) > 0$ and $w''(k) < 0$.

A young agent allocates a portion of her after-tax wage income to investment in physical capital and the remainder to investment in her own health. Each young agent born at date $t = 1$ maximizes

$$U(c_{t+1}^{t}) = \theta(x_t)u(c_{t+1}^{t})$$

subject to

$$x_t + s_t = (1-t)w_t \tag{4.5}$$

and

$$c_{t+1}^{t} = R_{t+1}s_t, \tag{4.6}$$

where s_t denote saving in the form of physical capital by a young agent and x_t denotes private investment in health by a young agent. Agents undertake current investment in health to achieve a longer lifespan in the future. They trade off this investment in their own health with contemporaneous investment in physical capital. Return to savings are made available to old agents right at the very beginning of the second period of their life.[14] The agent takes $w_t, R_{t+1}, \eta(p_t)$, and τ as given. Then the agent's problem is to choose x_t by solving

$$\max_{x_t} \theta(x_t)\frac{[R_{t+1}\{(1-\tau)w_t - x_t\}]^{1-\sigma}}{(1-\sigma)} \text{ subject to } \theta(x) \in (0,1].$$

Below we will focus on interior solutions to this problem (see footnote 14).

Lemma 1 *If*

$$0 < b\eta < \sigma, \tag{4.7}$$

then $\theta(x)u(c(x))$ is strictly concave in x, and the agent's problem has a unique solution.[15]

[14]This allows us to abstract away from issues relating to the eventual fate of the savings of those who die before receiving any return on them; see Chakraborty and Das (forthcoming) for details.

[15]*Assumption (4.7) is verified in everything that follows.*

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The first order condition to the agent's problem is given by[16]

$$\theta'(x_t)u(c_{t+1}^t) = \theta(x_t)u'(c_{t+1}^t)R_{t+1} \tag{4.8}$$

which easily reduce to[17]

$$\frac{\theta'(x_t)}{\theta(x_t)}(1-t)w_t - x_t = (1-\sigma). \tag{4.9}$$

Using (4.2), the optimal private health investment can be computed to be

$$x_t = \frac{(1-\tau)b\eta(p_t)}{1-\sigma+b\eta(p_t)}w_t, \tag{4.10}$$

and from (4.5), the optimal savings function is given by

$$s_t = \frac{(1-\tau)(1-\sigma)}{1-\sigma+b\eta(p_t)}w_t. \tag{4.11}$$

where $p_t = \tau w_t$.

A nice implication of our assumptions on preferences etc. is that agents' optimal saving in the form of physical capital is realistically independent of the rate of return on capital. Also from (4.10)-(4.11) it follows that ceteris paribus, longevity and youthful savings are both normal goods. As agents' incomes rise, they increase their saving both in the form of physical capital and in the form of health investments. Furthermore, an investigation of (4.10)-(4.11) reveals that ceteris paribus, private investments in health rises and optimal saving in capital falls as public expenditure on health rises.

4.4 General Equilibrium

4.4.1 Stationary Equilibria

In general equilibrium, using $s_t = k_{t+1}$ and (4.3), we can rewrite (4.11) as

$$k_{t+1} = \frac{(1-\tau)(1-\sigma)}{1-\sigma+b\eta(p_t)}w(k_t) \equiv g(k_t). \tag{4.12}$$

[16]For a general utility function u, the first order condition is given by $-\theta'(x^*)u(c^*)+\theta(x^*)u'(c^*)R = 0$. When u has the CRRA form, this reduces to (7).

[17]It is possible to check that x and c are substitutes (even gross substitutes). As a referee pointed out, this has the counterintuitive implication that agents may save (s) more to raise future consumption and yet reduce the length of time (θ) they get to enjoy that higher consumption. This is reminiscent of the tradeoff between quality and quantity of life studied in Dow, Philipson, and Sala-i-Martin (1999). There an agent may "allocate his wealth between consumption or life extension, [and] health investments allow the individual to convert quality of life into quantity of life." We thank an anonymous referee for suggesting this interpretation.

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Eq. (4.12) is the law of motion for the capital-labor ratio. Given a k_1 and τ, sequences $\{k_t\}_{t=2}^{\inf}$ that satisfy (4.12), and (4.7) with $\theta \in (0,1]$ and $b \in (0, \frac{1}{\eta})$ constitute valid dynamic competitive equilibria. Once this solution sequence is known, it is possible to compute equilibrium sequences $\{x_t\}_{t=1}^{\inf}$, $\{\theta_t\}_{t=1}^{\inf}$,$\{w_t\}_{t=1}^{\inf}$, etc.

For future reference, note that

$$\frac{dk_{t+1}}{dk_t} = \frac{(1-\tau)(1-\sigma)w'(k_t)}{1-\sigma+b\eta(p_t)}[1 - \frac{b\tau\eta'(p_t)w(k_t)}{1-\sigma+b\eta(p_t)}]. \tag{4.13}$$

Note if $\eta'(.) < 0$, then as is evident from(4.13), the law of motion in (4.12) represents an increasing relationship between the capital-labor ratios at two successive dates. In this case, as is well-known from Azariadis (1993; Ch. 8), there is no possibility for the time map $g(.)$ to exhibit "complex dynamics" or endogenous fluctuations of any kind.

Steady state equilibria are time-invariant solutions to (4.12). At a steady state, $k_{t+1} = k_t = k^*$. Then using $w(k) = (1-\alpha)Ak^\alpha$, equation (4.12) reduces to

$$k^* = \frac{A(1-\tau)(1-\sigma)(1-\alpha)(k^*)^\alpha}{1-\sigma+b\eta(A\tau(1-\alpha)(k^*)^\alpha)}. \tag{4.14}$$

It is immediately clear that $k^* = 0$ is a solution to (4.14). Define $H(k) \equiv A(1-\tau)(1-\sigma)(1-\alpha)(k^*)^{\alpha-1} - b\eta(A\tau(1-\alpha)(k^*)^\alpha)$. To find the non-trivial steady states, we rewrite (4.14) as $H(k) = 1-\sigma$.

Proposition 1 *(i) There exists a non-trivial steady state solution to (4.12). (ii) An additional sufficient condition for the existence of an unique non-trivial steady state* (k^*) *is* $\eta'(p) > 0$.

While existence of a non-trivial solution to (4.14) is guaranteed, uniqueness follows from assuming that the elasticity of longevity with respect to private investment in health is *increasing* in the size of the public health program. Note that the condition described in Proposition (1) is sufficient and by no means necessary. Henceforth, we will assume

$$\eta'(p) > 0 \tag{4.15}$$

holds. Under (4.15), it follows there is a unique non-trivial steady state k^* (no closed form expression for k^* is however possible to obtain).

We go on to study the equilibrium comparative static properties of k^*.

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Proposition 2

$$a)\ \frac{\partial k^*}{\partial \tau} < 0,$$

$$b)\ \frac{dp_t}{dt} = \begin{cases} \geq 0 & \textit{if}\ \tau \leq 1-\alpha \\ < 0 & \textit{if}\ \tau > 1-\alpha \end{cases} \tag{4.16}$$

As the tax rate rises, ceteris paribus private investment in health and capital investment both fall. This reduces future wages which further reduces capital investment. When it comes to tax revenue (public expenditures on health), there is a standard Laffer curve: when the tax rate is low (high) enough, a further rise in the tax rate raises (reduces) public expenditures on health.

For future reference, note that if $t = 0$, then $\eta(0) = 1$. As noted above, this is an economy in which the public component to longevity production is absent. It follows that for $t = 0$, (4.14) has a unique closed form solution given by

$$\tilde{k} = [\frac{A(1-\sigma)(1-\alpha)}{(1-\sigma)+b}]^{1/(1-\alpha)}. \tag{4.17}$$

It is also easy to check that $\tilde{k}$ is asymptotically stable. Thus, when there are no public expenditures on promoting longevity, the economy behaves like a standard Diamond (1965) economy and converges monotonically to a unique stationary level of real activity.[18] In other words, without the public component, the economy considered is immune to the possibility of endogenous fluctuations. To foreshadow, we show below that tax-financed public expenditures aimed at raising people's longevity may introduce endogenous volatility in an economy where such fluctuations are impossible in their absence.

4.4.2 Non-stationary Equilibria

4.4.2.1 Endogenous Fluctuations

We are now in a position to piece together more information about the law of motion (4.12) and non-stationary equilibria. We are particularly interested in the possibility that

[18] Along the transition, private investment in health (x) rises monotonically as does longevity (θ).

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the law of motion may produce non-stationary equilibria in which the economy undergoes endogenous fluctuations or more generally, complex dynamics. These fluctuations are interesting to economists because they represent stylized business cycles that are generated purely from within an economic system and not from exogenous stochastic shocks.

From (4.13), it is clear that if $\eta'(p) < 0$ holds, the law of motion (4.12) is monotonic. In that case, as is well known, no endogenous fluctuations are possible. In other words, (4.15) is a necessary (but not sufficient) condition for the possibility of endogenous fluctuations. The next result outlines further sufficient restrictions on $\eta(p)$ which in conjunction with (A.1) would constitute a set of *sufficient* conditions for the law of motion in (4.12) to be non-monotonic.

Proposition 3 *If along with (4.15),*

$$\eta''(.) > 0 \tag{4.18}$$

holds, then the law of motion in (4.12) is non-monotonic. Indeed, the time map $g(.)$ in (4.12) is unimodal.[19] [20]

Proposition 3 has the following message: if the $\eta(.)$ function is convex, then the economy at hand satisfies the necessary conditions for endogenous volatility. In other words, (4.15)-(4.18) constitute a set of sufficient conditions for the map (4.12) to be non-monotonic which, in turn, is a *necessary* condition for the generation of endogenous fluctuations.

Given the complicated nature of (4.12), it seems appropriate to elucidate the intuition behind the endogenous volatility by means of an example.

Example 1 *Suppose $\eta(p) = 1 + 0.5p^4$, $f(k) = 16k^{0.6}$, $\sigma = 0.9$, $\tau = 0.3$, and $b = 0.001$. Then, it is possible to verify that assumptions (4.7), (4.15) and (4.18) are verified, and the law of motion $g(.)$ has the configuration in Figure 1. Also, $k^* = 3.75$. Then $\frac{dk_{t+1}}{dk_t}|_{k^*} = -0.40$ implying the existence of endogenous fluctuations near k^*. The fluctutations in all endogenous variables of interest, k, p, θ, and x, are depicted in Figure 2.*

[19] *The maximal point $\hat{k}$ of the time map $g(.)$ is determined from the following equation $J(\hat{k}) \equiv b\tau\eta'(\tau w(\hat{k}))w(\hat{k}) - b\eta(\tau w(\hat{k})) = 1 - \sigma$.*

[20] *Condition (4.18) is merely a sufficient condition. It is easy to check that (4.7)-(4.18) are satisfied for $\eta(p) = a + dp^{c\xi}$, where a, d, c are all positive and $\xi > 1$.*

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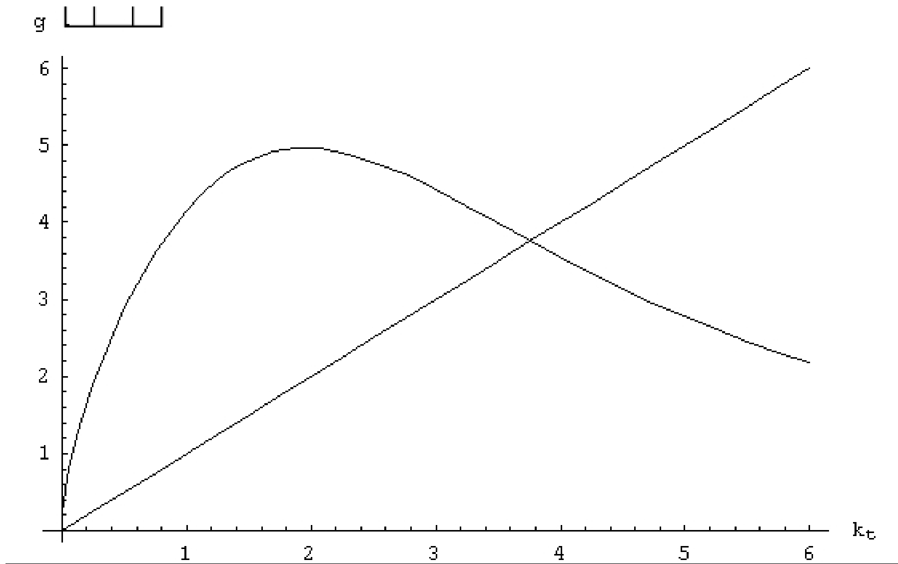

Figure 4.1 The time map $g(.)$ for Example 1

We now explore the intuition behind the endogenous fluctuations. Suppose the economy starts off at a relatively low level of k. Then, in that period, the wage income of the young will be relatively low. This will cause the government's tax revenue to be relatively low and hence, η, the public input in private longevity will be relatively low. As a result, because of the complementarity between the private and the public inputs in longevity, the private investment in longevity by the young will be relatively low; correspondingly, the investment in capital will be higher causing next period's capital-labor ratio to be relatively high. Figure 2 contains a diagrammatic exposition of this argument using the parametric specification in Example 1.

It bears emphasis that the fluctuations are entirely caused by the assumed complementarity between the private and the public inputs in longevity. To see this in a clean way, we now undertake a short *detour* to consider an alternative formulation for the function $\theta(x, \eta)$. Instead of the specification in (4.1), let us consider the CES form:

$$\theta(x, \eta) = (x^\rho + \eta^\rho)^{\frac{1}{\rho}} \tag{4.19}$$

where x and η are complements if $\rho < 0$ and substitutes if $\rho > 0$. It is easy to check that $\frac{\partial\theta(x,\eta)}{\partial x} > 0$ and $\frac{\partial\theta(x,\eta)}{\partial \eta} > 0$. Then using $\frac{\theta'(x)}{\theta(x)} = \frac{x^{\rho-1}}{(x^\rho+\eta^\rho)}$, we can check that (4.9) reduces to

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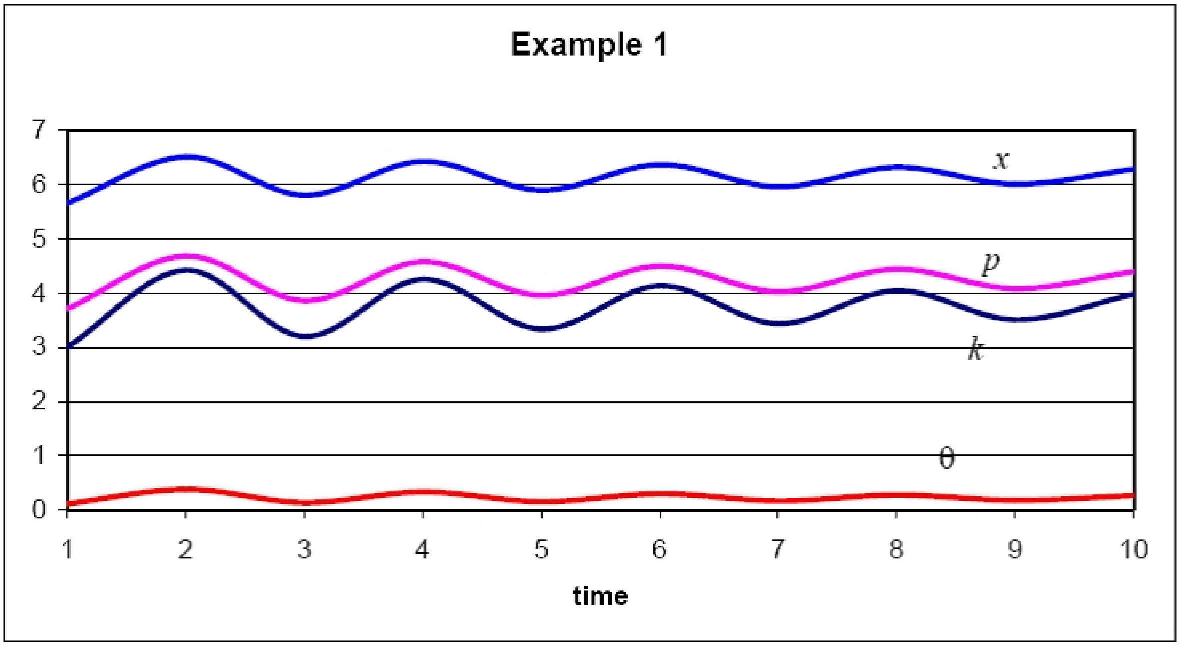

Figure 4.2 Endogenous fluctuations in Example 1

$$(\frac{x_t^\rho}{(x_t^\rho + \eta_t^\rho)})\frac{[(1-\tau)w_t - x_t]}{x_t} = (1-\sigma). \tag{4.20}$$

Define $y_t(k_t) \equiv y_t \equiv \frac{1}{x_t}$, then it follows (after some algebra) that (4.20) yields

$$(1-\sigma)(\eta_t y_t)^{rho} - (1-\tau)w_t y_t + (2-\sigma) = 0. \tag{4.21}$$

Straightforward differentiation of (4.21) along with some rearrangement produces

$$\frac{y_t'(k_t)}{(y_t(k_t))^2} = -w'(k_t)[\frac{(1-\tau) - (1-\sigma)\rho(\eta_t y_t)^{\rho-1}\eta'(p_t)\tau}{(2-\sigma) + (1-\sigma)(1-\rho)(\eta_t y_t)^\rho}]. \tag{4.22}$$

Since $x_t + s_t = (1-\tau)w_t$, we have

$$k_{t+1} = (1-\tau)w_t - x_t = (1-\tau)w(k_t) - \frac{1}{y_t(k_t)}.$$

It follows that

$$\frac{dk_{t+1}}{dk_t} = (1-\tau)w'(k_t) + \frac{1}{[y_t(k_t)]^2}y_t'(k_t). \tag{4.23}$$

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Using (4.21) we can write (4.23) as

$$\frac{dk_{t+1}}{dk_t} = (1-\tau)w'(k_t) - w'(k_t)[\frac{(1-\tau)-(1-\sigma)\rho(\eta_t y_t)^{\rho-1}\eta'(p_t)\tau}{(2-\sigma)+(1-\sigma)(1-\rho)(\eta_t y_t)^{\rho}}]. \tag{4.24}$$

Routine algebra verifies that (4.24) reduces to

$$\frac{dk_{t+1}}{dk_t} = \frac{w'(k_t)(1-\sigma)\{(1-\tau)+(1-\tau)(1-\rho)(\eta_t y_t)^{\rho}+\rho(\eta_t y_t)^{\rho-1}\eta'(p_t)\tau\}}{(2-\sigma)+(1-\sigma)(1-\rho)(\eta_t y_t)^{\rho}}.$$

Private and public inputs to longevity (i.e., x and η) are complements when $\rho < 0$. Since $\sigma < 1$, it follows that a necessary (but not sufficient) condition for cycles (i.e., $\frac{dk_{t+1}}{dk_t} < 0$) is $\rho < 0$. The upshot is that for the economy at hand to generate endogenous volatility, it is necessary that the public and private inputs to agents' longevity be complements.

4.4.2.2 Complex Dynamics

Chaotic equilibria are characterized by the feature that they are extremely sensitive to the specification of the initial conditions. In other words, chaotic equilibria represent equilibrium "solutions that start in the same small neighborhood (and) lose all resemblance to each other after a few iterations" (Azariadis (1993), p. 106). Thus economies whose equilibria exhibit chaotic properties may start off "looking very similar" and end up "looking very different". Such a statement potentially addresses the puzzle raised in Lucas (1993) of why Korea and the Phillipines with very similar initial conditions in 1960 went on to experience very different levels of development over the next three decades.

To explore the possibility of complex dynamics (specifically, aperiodic and chaotic behavior), we have to go beyond the necessary conditions outlined in Proposition 3 and seek sufficient conditions for such behavior to arise. Mitra (2001) offers one set of sufficient conditions for chaos in unimodal maps (like g). Mitra (2001) focuses solely on dynamical systems (X, g), where the state space X is an interval on the non-negative part of the real line. The map, g, is required to be a continuous function from X to X, unimodal with a maximum at $\hat{k}$ with $g(\hat{k}) > k$, and the unique steady state (k^*) must satisfy $k^* > \hat{k}$. For such maps, Mitra (2001) states the following theorem (his Proposition 2.3, pg. 142) which we restate for the sake of completeness.

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Theorem 1 (Mitra, 2001) *Let (X, g) be a dynamical system. If g satisfies $g^2(\hat{k}) < \hat{k}$ and $g^3(\hat{k}) < k^*$, then (X, g) exhibits topological chaos.*

As mentioned earlier, it is not possible to produce closed-form expressions for $\hat{k}$ and k^*. As such, it is not feasible to produce a set of simple parametric conditions satisfying all of Mitra's sufficient conditions for chaos. Below we verify by means of a numerical example that there exist time maps $g(.)$ defined in (4.12) that satisfy these conditions.

Example 2 *Suppose $\eta(p) = 1 + 0.5p^8$, $f(k) = 20k^{0.6}$, $\sigma = 0.9$, $\tau = 0.21$, and $b = 0.001$. Then, it is possible to verify that assumptions (4.7), (4.15) and (4.18) are verified. Also, $\hat{k} = 0.84$, $k^* = 1.70$, $g^2(\hat{k}) = 0.02$, and $g^3(\hat{k}) = 0.66$ implying the conditions in the theorem above are satisfied. Hence, for this parametric specification, the time map $g(.)$ in (4.12) exhibits topological chaos.*[21] [22]

To understand the significance of this result, imagine two economies that differ only in their initial conditions. One may approach k^* monotonically; while the other may get "infected" by cyclical fluctuations in all its endogenous variables, sometimes coming "very close" to the steady state k^*, but eventually being repelled away. Such an economy will not be able to sustain a permanently high level of real activity or a high level of life expectancy for its citizens.

It is important to point out that the nonmonotonicity of $g(.)$ [a necessary condition for existence of periodic and chaotic equilibria] is entirely a consequence of the interaction of private and public investments in health. To see this, note that in the absence of the public input, we can write $\theta(x) = bx^b$ [with $\tau = 0$] and hence (4.12) would be replaced by $k_{t+1} = \frac{1-\sigma}{1-\sigma+b}w(k_t)$. Since $w'(.) > 0$, it is easy to see that this would not produce a non-monotonic time map. The message is clear: tax-financed public expenditures aimed at complementing private efforts to increase longevity may introduce endogenous volatility in an economy where such fluctuations are otherwise impossible in their absence.

[21] *It is also easy to check that for this parametric specification, $\theta(k^*) = 0.91$.*

[22] *A few words about the realism of the above example are in order. As has been argued (see Chakraborty, 2004) a may easily be rationalized to be above 0.5 simply by broadening the concept of capital to include human capital. The above parametric specification produces a range of $\theta \in (0.4, 0.9)$. This is in line with cross country WHO data suggesting that the range of ages people survive after age 60 is 7-18 years. The choice of σ follows standard practice in the mortality literature, e.g., Shepard and Zeckhauser (1984). In passing, also note that examples similar to Example 2 are easy to generate.*

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4.5 Conclusion

Health systems comprise two wings, a public and a private one. A major goal of the public wing (public health programs) is to promote healthy behavior. Of course, such public efforts to improve health, longevity, and well-being need private backing too. This paper presents a simple framework to study this dynamic complementarity between public health programs and private efforts to improve health and longevity. More specifically, it introduces endogenous longevity in an otherwise standard overlapping generations model with capital. In the model, an agent may privately invest in her own longevity by incurring expenses funded from her wage income. Such private health investments are more "productive" if accompanied by complementary tax-financed public health programs. We find that the public input in private longevity may expose the economy to aggregate chaotic fluctuations otherwise not possible in the absence of the public input.

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4.7 Appendix

4.7.1 Proof of Lemma 1

A sufficient condition for $\theta(x)u(c)$ to be concave is that its Hessian

$$\begin{vmatrix} \theta''(x)u(c) & \theta'(x)u'(c) \\ \theta'(x)u'(c) & \theta(x)u''(c) \end{vmatrix}$$

be negative definite, i.e., a) $\theta''(x)u(c) < 0$, and b) $\theta''(x)u(c)\theta(x)u''(c) - (\theta'(x)u'(c))^2 > 0$. The assumed concavity of θ ensures (a). To ensure (b), we require

$$\theta''(x)u(c)\theta(x)u''(c) > (\theta'(x)u'(c))^{2.} \tag{4.25}$$

Using the specified utility function and longevity function, (4.25) reduces to

$$b^2\eta^2(b\eta - 1)x^{b\eta-2}b\eta x^{b\eta}((c^{1-\sigma})/(1-\sigma))(-\sigma)c^{-\sigma-1} > b4\eta4x^{2b\eta-2}c^{-2\sigma},$$

which further reduces to $b\eta < \sigma$.

4.7.2 Proof of Proposition 1

Solutions to $H(k) = 1 - \sigma$ determine non-trivial steady state capital-labor ratios. Since $\eta(p) > 0$, it is easy to check that

$$\begin{aligned} H(0) &= \lim_{k\to 0} H(k) = \infty - b\eta(0) = \infty \\ H(\infty) &= \lim_{k\to\infty} H(k) = 0 - b\eta(\infty) < 0 \end{aligned}$$

holds. Since $H(\cdot)$ is a continuous function, there exists a k^* such that $H(k^*) = 1-\sigma$. If $H(\cdot)$ is monotonically decreasing, then there must exist one and only one non-trivial steady solution. Since

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$$H'(k) = -A(1-\tau)(1-\sigma)(1-\alpha)^2(k^*)^{\alpha-2} - A\alpha b\tau(1-\alpha)(k^*)^{\alpha-1}\eta'(A\tau(1-\alpha)(k^*)^{\alpha}),$$

it follows that if $\eta'(p) > 0$, then $H'(k) < 0$.

4.7.3 Proof of Proposition 2

a) Straightforward differentiation of $H(k) = 1-\sigma$ reveals

$$\frac{\partial k^*}{\partial \tau} = -\frac{A(1-\sigma)(1-\alpha)(k^*)^{\alpha-1} + A(1-\alpha)b\eta'(\tau w(k^*))(k^*)^{\alpha}}{A(1-\tau)(1-\sigma)(1-\alpha)^2(k^*)^{\alpha-2} + A\tau\alpha(1-\alpha)b\eta'(\tau w(k^*))(k^*)^{\alpha-1}} \tag{4.26}$$

Using (4.26), the result follows.

b) It follows from the definition of $p(.)$ that the sign of $\frac{\partial p(.)}{\partial \tau}$ is the same as the sign of $(k^* + \tau\alpha\frac{\partial k^*}{\partial \tau})$. Using (4.26), one can verify that

$$\text{sign}(k^* + \tau\alpha\frac{\partial k^*}{\partial \tau}) = \text{sign}\frac{(1-\sigma)(1-\tau-\alpha)}{(1-\tau)(1-\alpha)(1-\sigma) + \tau\alpha b\eta'(\tau w(k^*))k^*}k^*$$

It follows that if $\tau \leq 1-\alpha$, then $\frac{\partial p(.)}{\partial \tau} = 0$.

4.7.4 Proof of Proposition 3

From (4.13), it follows that

$$\begin{aligned} \text{sign}(\frac{dk_{t+1}}{dk_t}) &= \text{sign}[1 - \frac{b\tau\eta'(\tau w(k_t))w(k_t)}{1-\sigma+b\eta(\tau w(k_t))}] \\ &= \text{sign}[1-\sigma+b\eta(\tau w(k_t)) - b\tau'(\tau w(k_t))w(k_t)] \end{aligned}$$

Let $J(k) \equiv b\tau\eta'(\tau w(k_t))w(k_t) - b\eta(\tau w(k_t))$. Since $\eta'(0) < \infty$, it follows that $J(0) < 0$. Also,

$$\begin{aligned} J'(k) &= b\tau\eta''(\tau w(k))\tau w'(k)w(k) + b\tau\eta'(\tau w(k))w'(k) - b\tau\eta'(\tau w(k))w'(k) \\ &= b\tau\eta''(\tau w(k))\tau w'(k)w(k) > 0 \end{aligned}$$

Note that $J(k)$ may be rewritten as $J(p) = b\eta'(p)p - b\eta(p)$. Using (4.18), it follows that $\eta'(p)p$ grows faster than $\eta(p)$, and so $\lim_{k\to\infty} J(k) = \infty$. Therefore, there exists a unique $\hat{k}$ such that $J(\hat{k}) = 1-\sigma$. Hence, when $k < \hat{k}$, $J(\hat{k}) < 1-\sigma$ and when $k > \hat{k}$, $J(\hat{k}) > 1-\sigma$.

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CHAPTER 5. GENERAL CONCLUSION

This dissertation contains three papers. The first paper studies how the existing UI program affects a firm's decision to layoff workers in a model with moral hazard problems. The calibrated solution shows that there are two types of layoffs. Workers who are poor due to a sequence of bad outputs are laid off since they are too poor to be punished. Rich workers who have experience a long sequence of good outputs are also fired because they are too rich to be motivated. I then remove the existing UI system and compare the equilibrium outcomes to the calibrated solution. This policy experiment suggests that the existing UI system induces more layoffs. This is because the UI system affects the economy in several ways. First, the unemployment benefit reduces the layoff cost. Hence, firms tend to terminate workers more often. Second, the UI tax has a distortionary effect on the type of workers laid off. Since rich workers choose not to go back to the labor market, they are not eligible for receiving UI benefits. However, firms that hire these workers still have to pay for the UI tax, which further reduces their profits. Therefore, rich workers are more likely to be laid off. In the equilibrium I find that the existing UI system reduces the unemployment rate, contrary to the previous results when the focus is on the workers.

The second paper evaluate the effects of the existing UI program. Two features make this paper distinct from the previous work. First, agents are able to borrow and lend to finance their consumption; second, the government is able to hold a non-zero balanced budget. There are two major findings. The existing UI program reduces the welfare comparing to the case when agents purely rely on the self-insurance scheme against the unemployment risk, which is opposite to most previous findings. It shows that the self-insurance scheme does matter in measuring the UI effects. The second finding is that the current positive UITF balance improves

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the welfare in the presence of a UI program. This latter finding suggests that the government is potentially able to set an optimal UITF balance. However, as mentioned earlier, this policy implication needs special cautions. The effects of the UITF balance may be underestimated if we take into account that the balance in the UITF account can be used to pay for increasing claims due to a negative aggregate shock, or recession. This can be a future extension of the current paper.

The last paper considers the relation between longevity and the economics of the health systems. The upshot is that the tax-financed public health expenditures may introduce endogenous volatility in an economy when it is aimed at improving an individual agent's longevity. Nevertheless, such fluctuation would never appear in the absence of the public health expenditures. This finding provides a possible explaination for the many dramatical reversals in life expectancy especially in Sub Saharan Africa and former Eastern Bloc nations as noted by the WHO Commission on Macroeconomics and Health.

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